Missouri K-12 Mathematics:

Core Concepts, Learning Goals and Performance Indicators[1]

DRAFT

March 28, 2008

Missouri Department of Elementary and Secondary Education

Mathematics, Engineering, Technology and Science (METS) Alliance

[1] The final draft of this document will be completed by the Fall of 2008 and will be used to inform and update GLEs and CLEs as required by the Missouri Assessment Program.

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Introduction

Since the Outstanding Schools Act of 1993, several documents have been developed to aid Missouri school districts in creating curriculum enabling all students to achieve their potential. Most prominently, the *Show-Me Standards* (1996) identify broad content knowledge and process skills for all students to be successful as they continue their education, enter the workforce, and assume civic responsibilities. Shortly after, the *Framework for Curriculum Development* was published to provide Missouri school districts with a "frame" for building curricula using the *Show-Me Standards* as a foundation.

In response to the federal No Child Left Behind Act of 2001, the Missouri Department of Elementary and Secondary Education created the first set of K-12 grade-level mathematics learning expectations (GLEs), published in 2004. This document, updated to version 2.0 in 2007, conveyed to Missouri educators and to test developers mathematics assessment specifications for specific grades, K-12. That is, it conveyed the mathematics that would be the focus of state assessment and the mathematics that would be assessed locally. However, it was not developed to specify all the mathematics content that should be taught at each grade level or to what extent specific mathematics content should be emphasized.

In 2007 three factors led to the development of this new document. First, Governor Matt Blunt hosted a summit of key business stakeholders focused on ensuring that Missouri would continue to compete in the global market calling for increased skills in the areas of mathematics, engineering, technology, and science. A recommendation of Summit participants called for a revision of Missouri's K-12 GLEs and assessments for mathematics and science to support instruction of important mathematics content and process skills (problem solving, reasoning, communication, connections, and representations).

Second, the Missouri State Board of Education authorized a change in the required high school annual assessment – moving from a general grade 10 mathematics assessment to a series of end-of-course assessments in mathematics, beginning with Algebra I in 2008-09. This change necessitated a revision of the 2004 Missouri GLE document from a grade-based framework to a course-based framework at the high school level.

Third, national attention continued to focus on strategies to strengthen K-12 mathematics programs. In particular, there is a commonly held belief that the U.S. mathematics curriculum lacks focus, coherence and depth (e.g., "mile wide, inch deep" phenonomen characterized by the Trends in International Mathematics and Science Study). In response to this concern, several national organizations developed guidelines for mathematics including Achieve, Inc., the College Board, the National Council of Teachers of Mathematics and the American Statistical Association. In addition, the National Assessment of Educational Progress created a new framework for test development.

These factors led to collaboration between the Mathematics, Engineering, Technology and Science (METS) Alliance and the Missouri Department of Elementary and Secondary Education (DESE). As a joint initiative, METS and DESE collaborated to develop a comprehensive K-HS mathematics curriculum document that specifies content priorities (core concepts), learning goals and performance indicators for each grade (K-8) and core high school courses (Algebra 1 and 2, Geometry, and Integrated Mathematics 1, 2, and 3). This draft document is the result of that collaboration.

Overview of the Project

Beginning in November 2007 a writing group composed of Missouri mathematics teachers, curriculum specialists and higher education representatives, reviewed several national publications by groups including Achieve, Inc., the College Board, the American Statistical Association, and the National Council

of Teachers of Mathematics. Each national group made recommendations about mathematics curriculum emphasis and organization based on extensive analysis of the needs of today's workforce and preparation for college. Together with the Missouri GLEs (v. 2.0) and the NAEP framework, these documents were used to craft the curriculum recommendations in this document.

A primary goal of the work was to articulate a focused, coherent, and forward-looking mathematics program to prepare all Missouri students for careers and livelihood in today's technology- and statistics-rich environment. Care was taken to outline a few content priorities (core concepts) per grade level or course rather than attempt to cover a wide variety of topics across all mathematics strands. However, mathematics from each strand (number/algebra; geometry/measurement; and data analysis/statistics) was given attention across K-12. In some cases (e.g., algebra in middle school) the strand is given priority while in other cases (e.g., data analysis/statistics in grade 1) the strand is not a content priority but can be used to support the development of other strands (e.g, number/algebra).

In addition to specification of content emphasis, this document emphasizes the development of important mathematical processes such as problem solving, reasoning, communication, connections and representations. Without these processes, the skills and understandings outlined in the content strands are of little value.

The writing group also considered a report by a committee of the National Research Council. In the report (*Adding It Up: Helping Children Learn Mathematics*), the committee emphasized the importance of "mathematical proficiency" for all students. This term conveys what it means for someone to learn mathematics successfully. It includes five interwoven and interdependent strands:

- *conceptual understanding*—comprehension of mathematical concepts, operations, and relations.
- *procedural fluency*—skill in carrying out procedures flexibly, accurately, efficiently, and appropriately
- *strategic competence*—ability to formulate, represent, and solve mathematical problems
- *adaptive reasoning*—capacity for logical thought, reflection, explanation, and justification
- *productive disposition*—habitual inclination to see mathematics as sensible, useful, worthwhile, coupled with a belief in diligence and one's own efficacy. (NRC, 2001, p. 116).

Writing Group

This document was developed by a team of Missouri educators (K-12 teachers, curriculum supervisors, and representatives of higher education) with assistance from support staff from DESE and METS. The individuals who played key roles in this project include:

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Overview of the Document

This document presents a draft of curriculum priorities (core concepts), learning goals and performance indicators for K-12 mathematics in Missouri schools. The draft is based on several principles adopted by the writing group:

- Consider the mathematics all Missouri students should learn.
- Make clear the importance of all aspects of mathematics: conceptual understanding, skills and mathematical processes, with mathematics proficiency (National Research Council, 2001) as the primary goal.
- Identify those topics that should be taught for extended periods of time at each grade level and show how topics develop over grade levels.
- Strive for clarity, specificity, and measurability of the performance indicators (assessment specifications)

This document outlines important mathematics ideas, concepts, skills and procedures that form the foundation for understanding and learning the subject of mathematics. It provides a framework to bring focus to teaching, learning, and assessing mathematics.

Emphasis on Mathematical Content

The DRAFT document is formatted differently form prior Missouri GLE documents. Rather than a matrix format (organized by grade and strand), this document is organized by core concepts (content priorities) at each grade and course, followed by learning goals and then by performance indicators.

> *Core Concepts* – A few important mathematical ideas for a grade level or course that serve as organizing structures for curriculum design, instruction, and assessment.
>
> *Learning Goals* - A set of statements, organized around each core concept, indicating what students are expected to learn.

Performance Indicators - Statements of specific and measurable learning outcomes.

The core concepts at each grade level (K-8) and high school course represent a critical and unique element of this revision. They describe important mathematics to be learned and show how the individual goals and performance expectations fit together within and across grade levels to build a cohesive curriculum.

Performance Indicators have been reviewed to identify candidates for assessment at the state or local level. An item coded with an asterisk (*) should be assessed at the local level. An item with no asterisk will be assessed at the state level (Grade 3 – 8 MAP Assessment or End-of-Course Exams).

Emphasis on Mathematical Processes

At every grade, students must be challenged to use mathematics to reason and solve problems and they must be provided opportunities to develop these mathematical processes. In addition, they must be able to communicate about mathematics including representing (in writing, speaking and various mathematical formats) mathematical ideas.

The National Council of Teachers of Mathematics (2000) describes five mathematical processes that are essential elements of K-12 mathematics. They include:

Problem Solving: Students will develop their Problem Solving ability by engaging in developmentally appropriate problem-solving opportunities in which there is a need to use various approaches to investigate and understand mathematical concepts; to formulate their own problems; to find solutions to problems from everyday situations; to develop and apply strategies to solve a wide variety of problems; and to integrate mathematical reasoning, communication and connections.

Reasoning and Proof: Students will develop their Reasoning and Proof ability by solving problems in which there is a need to investigate significant mathematical ideas in all content areas; to justify their thinking; to reinforce and extend their logical reasoning abilities; to reflect on and clarify their own thinking; to ask questions to extend their thinking; and to construct their own learning.

Communication: Students will develop their mathematical Communication ability by solving problems in which there is a need to obtain information from the real world through reading, listening and observing; to translate this information into mathematical language and symbols; to process this information mathematically; and to present results in written, oral, and visual formats.

Connections: Students will develop mathematical Connections by solving problems in which there is a need to view mathematics as an integrated whole and to integrate mathematics with other disciplines, while allowing the flexibility to approach problems, from within and outside mathematics, in a variety of ways.

Representations: Students will create and use representations to organize, record, and communicate mathematical ideas. They select, apply, and translate among mathematical representations to solve problems. They use representations to model and interpret physical, social, and mathematical phenomena.

In this document mathematical processes are interwoven throughout the core concepts and learning goals so that they are not thought of in isolation but rather as a part of learning the content of mathematics. Key ideas related to mathematical processes are also listed in the "Core Concept

Matrix" (see pp. 8-10) by grade band.

Role of Technology

The writing group is unanimous in its support of use of technology to increase opportunities for student learning, enrich learning environments, and prepare students for an increasingly technology-rich work and life environment. While we agree that some of the mathematics outlined in this document should be developed without the aid of technology, much of the mathematics can be viewed and explored in new ways with the benefit of technology.

The content priorities, learning goals and performance indicators describe the mathematics students need to learn, not necessarily all of the tools that teachers might use to help students reach these goals. Teachers need to be knowledgeable about how technology can support students in learning mathematics, not replace their learning. More importantly, teachers need to know what mathematics students need to understand so that the students can not only use the technology, but also can make sense of the outputs from that technology. We trust that teachers, with the help of curriculum specialists, will make appropriate choices regarding the use of technology. In the work outlined in this document, the writing group has taken into consideration the availability of technology. However, we have not prescribed how teachers should organize or enact instruction. Teachers are best qualified for those decisions.

Relationship to Recommendations of National Groups and Other DESE Documents

The grade and course level core concepts, learning goals and performance indicators are built upon the foundation of the *Show-Me Content and Process Standards* (1996). Other important resources in the development of this draft include the National Assessment of Educational Progress framework, National Council of Teachers of Mathematics *Principles and Standards for School Mathematics* (2000) and *Curriculum Focal Points* (2006), Achieve, Inc. *Standards for Elementary and Secondary*, and the American Statistical Association *Guidelines for Assessment and Instruction in Statistics Education* (2006). In addition, the 2007 draft of Missouri GLEs and the Department of Higher Education College Entrance Competencies (2008) were key resources.

Summary

The writers involved with the development of this document believe that every Missouri student should have access to a high-quality mathematics education. The mathematics content priorities, learning goals and performance indicators outlined here are presented in the spirit that they will be taught to and achieved by every Missouri student.

Feedback on the Draft is Encouraged

This document is a draft and as such, is intended to stimulate review and discussion. Feedback is encouraged. The writing group will consider all feedback in producing the final draft to be submitted to the State Board of Education in September 2008 (see http://dese.mo.gov/divimprove/curriculum/unitindex.html for options for providing feedback).

Summary of Core Concepts, Kindergarten Through High School, by Mathematics Content Strand

	Number/Algebra	Geometry/Measurement	Data/Probability	Problem Solving/Reasoning Communication/Connections/Representation
K	Develop an understanding of whole numbers, number patterns and relationships, and of the concepts underlying counting. Develop an understanding of the relationships among whole numbers and of joining and separating sets.	Develop an awareness of geometric shapes and spatial relationships. Develop an understanding of measurable attributes and the informal process of measurement to compare and order objects.		Use a variety of strategies to solve problems. Make and investigate mathematical conjectures. Communicate mathematical thinking coherently and clearly to peers and teacher. Recognize and apply mathematics in contexts outside of mathematics. Create and use representations to organize, record, and communicate mathematical ideas. Select, apply, and translate among mathematical representations to solve problems.
Gr. 1	Develop an understanding of whole number patterns and relationships, including grouping in tens and ones. Develop an understanding of addition and subtraction, apply a variety of strategies for basic addition and subtraction facts, and relate addition and subtraction as inverse operations.	Develop an understanding of the relationships among geometric shapes.		
Gr. 2	Develop an understanding of the base-ten numeration system and place-value concepts. Develop quick recall for basic related addition/subtraction facts, demonstrate fluency with multi-digit addition and subtraction strategies, and explain why strategies work on the basis of place value and properties of operations.	Develop an understanding of linear measurement and facility in measuring lengths.	Develop an initial understanding of data analysis by formulating questions and simple experiments, collecting, representing, analyzing, and interpreting data.	
Gr. 3	Develop an initial understanding of multiplication and division, relate multiplication and division as inverse operations, and apply a variety of strategies for basic related multiplication/division facts. Develop an understanding of fractions and fraction equivalence by using area, length, and set models.	Develop an understanding of properties and relationships in two-dimensional space. Develop an understanding of the relationships within measurement systems of time and money.		Apply and adapt a variety of strategies to solve problems. Monitor and reflect on the process of mathematical problem solving. Make and investigate mathematical conjectures. Communicate mathematical thinking coherently and clearly to peers and teacher. Analyze and evaluate the mathematical thinking and strategies of others. Create and use representations to organize, record, communicate, and model mathematical ideas. Select, apply, and translate among mathematical representations to solve problems.
Gr. 4	Develop quick recall for basic related multiplication/division facts, demonstrate fluency with whole number multiplication strategies, and explain why strategies work on the basis of place value and properties of operations. Develop understanding of decimals, including the connections between fractions and decimals.	Develop an understanding of attributes of two-dimensional shapes by exploring angles, angle classifications, area, and area measurement.	Develop an understanding of data analysis by formulating questions, collecting or using available data, analyzing tools of data representation, and using measures of center and spread to interpret data.	
Gr. 5	Develop an understanding of and fluency with whole number division strategies and explain why strategies work on the basis of place value and properties of operations. Develop an understanding of and fluency with addition and subtraction strategies for fractions (including mixed numbers) and decimals and explain why strategies work on the basis of place value and properties of operations.	Develop an understanding of the properties of three-dimensional shapes, including volume and surface area.	Develop an understanding of probability through the context of simple experiments and their outcomes.	

Summary of Core Concepts, Kindergarten Through High School, by Mathematics Content Strand

	Number/Algebra	Geometry/Measurement	Data/Probability	Problem Solving/Reasoning Communication/Connections/Representation
Gr. 6	Develop an understanding of and fluency with multiplication and division of rational numbers and extend this understanding to ratios and rates. Explore, represent, and generalize patterns.	Analyze properties of and measure two-dimensional shapes.	Design and conduct sample surveys, explore random selection, and compare data distributions.	Apply and adapt a variety of strategies to solve problems. Recognize reasoning and proof as fundamental aspects of mathematics. Make and investigate mathematical conjectures. Select and use various types of reasoning and methods of proof. Communicate mathematical thinking coherently and clearly to peers, teachers, and others. Evaluate the mathematical thinking and strategies of others. Use the language of mathematics to express mathematical ideas precisely. Create, select, apply, and translate among mathematical representations to solve problems.
Gr. 7	Develop an understanding of and fluency with integers and their operations. Develop and apply an understanding of proportionality. Identify, represent and solve linear patterns and relationships.	Develop and apply an understanding of transformations.	Estimate probabilities in experiments, comparing experimental and theoretical probabilities.	
Gr. 8	Represent and solve linear relationships and inequalities Develop understanding of real numbers, roots, and exponents.	Understand and apply the Pythagorean Theorem and measurement of three-dimensional figures.	Organize and summarize bivariate data, represent and quantify associations between two variables using simple models.	

Summary of Core Concepts, Kindergarten Through High School, by Mathematics Content Strand

Course	Number/Algebra	Geometry/Measurement	Data/Probability	Problem Solving/Reasoning Communication/Connections/Representation
Alg. 1	Interpret, model, justify and compare linear and nonlinear relationships. Interpret, represent and solve linear equations and inequalities. Model, interpret, represent and solve nonlinear functions.		Design statistical studies, recognize the importance of random sampling, identify sources of bias, and quantify the strength of linear relationships.	Create, apply and adapt a variety of strategies to solve problems. Recognize reasoning and proof as fundamental aspects of mathematics. Make and investigate mathematical conjectures. Develop and evaluate mathematical arguments and proofs. Use the language of mathematics to express mathematical ideas precisely. Understand how mathematical ideas interconnect and build on one another to produce a coherent whole. Create and use representations to organize, record, and communicate mathematical ideas Select, apply, and translate among mathematical representations to solve problems and model situations.
Geo.		Apply the concepts of inductive and deductive reasoning to form or verify conjectures. Apply and analyze transformations of figures in the coordinate plane using properties of similarity and congruency. Justify and apply measurement formulas for two- and three-dimensional shapes including the Pythagorean Theorem and its applications.	Explore two-stage experiments, conditional probability and independence and simulations of random phenomena.	
Alg. 2	Represent, compare, and translate polynomial equations and functions. Model, represent and solve non-linear functions (exponential, logarithmic, and other functions). Model, construct and solve systems of equations and inequalities.		Distinguish among experiments, surveys, and observational studies. Use permutations, combinations, and probability distributions to solve problems.	

Int. 1	Interpret, model, justify and compare linear and nonlinear relationships. Interpret, represent and solve linear equations and inequalities. Model, interpret, represent and solve nonlinear functions.	Represent, develop and apply patterns of shape, geometric reasoning, and geometric relationships.	Design statistical studies, recognize the importance of random sampling, identify sources of bias, and quantify the strength of linear relationships.	Create, apply and adapt a variety of strategies to solve problems. Recognize reasoning and proof as fundamental aspects of mathematics. Make and investigate mathematical conjectures. Develop and evaluate mathematical arguments and proofs. Use the language of mathematics to express mathematical ideas precisely. Understand how mathematical ideas interconnect and build on one another to produce a coherent whole. Create and use representations to organize, record, and communicate mathematical ideas Select, apply, and translate among mathematical representations to solve problems and model situations.
Int. 2	Model and solve systems of linear equations and matrices.	Develop and apply geometric proof, Ssmilarity, and transformations. Justify and apply measurement formulas for two- and three-dimensional shapes.	Explore two-stage experiments, conditional probability and independence, and simulations of random phenomena.	
Int. 3	Represent, compare, and translate polynomial equations and functions. Model, represent and solve non-linear functions (exponential, logarithmic, and other functions). Develop, analyze and model sequences and recursion.	Apply the Pythagorean Theorem and trigonometric ratios to indirect measurement situations.	Distinguish among experiments, surveys, and observational studies. Use permutations, combinations, and probability distributions to solve problems.	

KINDERGARTEN

Kindergarten, Core Concept A:
Develop an understanding of whole numbers, number patterns and relationships, and the concepts underlying counting.

Children develop an understanding of the meanings of whole numbers and recognize the number of objects in small groups with and without counting—the first most basic mathematical algorithm. They understand that number words refer to quantity. They use one-to-one correspondence to solve problems by matching sets and comparing number amounts. They use strategies for keeping track of what is counted, realize that the last word they state in counting tells "how many," and understand that rearrangement of objects in a set does not change the number of objects. They explore simple growing patterns as preparation for creating rules that describe relationships.

Learning Goals and Performance Indicators:

1) Understand and apply the counting process to various situations.

   a) *Count forward to 31 and backward from 10.
   b) *Use one-to-one correspondence when counting a set of objects and use a strategy to keep track of counted and uncounted objects
   c) *Know that the last counting word is the number in the collection
   d) *Recognize that the rearrangement of objects in a set does not change the number of objects.

2) Understand and use simple patterns as a foundation for describing relationships among whole numbers up to 31.

   a) *Describe, extend, generalize and create simple repeating patterns, recognizing that some patterns may have many possible repeating "units."
   b) *Identify and describe a possible repeating "unit" of simple repeating patterns and describe how these patterns are generated.
   c) *Apply the 0-9 sequence when writing numerals from one decade to the next decade.

** Locally assessed items.*

Kindergarten, Core Concept B:
Develop an understanding of the relationships among whole numbers and of joining and separating sets.

Children use numbers, including written numerals, to solve quantitative problems. They compare and order sets or numerals by using language such as "more than" and "less than" and by using both cardinal and ordinal meanings. They model simple joining and separating situations with objects. They choose, combine, and apply effective strategies for answering quantitative questions, including quickly recognizing the number in a small set, counting and producing sets of given sizes, counting the number in combined sets, and counting backward.

Learning Goals and Performance Indicators:

1) Understand the numbers occurring later in a number sequence are larger than those at the beginning of the number sequence by working with numbers at least through 31.

    a) *Translate among a variety of whole number representations, including objects, diagrams, words, and numerals.
    b) *Compare sets of objects and determine whether they have the same, fewer, or more objects.
    c) *Solve a variety of problems that involve comparing and ordering quantities.

2) Understand the meaning of addition and subtraction for whole numbers less than ten.

    a) *Represent the joining and separating of sets by using objects, diagrams, words, and numerals.
    b) *Compose and decompose quantities in order to establish relationship between the parts and the whole.
    c) *Recognize 5 or 10 as a part of the part-whole relationship of larger numbers.
    d) *Use the strategies of counting up and counting backward to solve addition and subtraction problems.
    e) *Use objects and drawings to create and solve simple problems involving addition or subtraction.

* *Locally assessed items.*

Kindergarten, Core Concept C:
Develop an awareness of geometric shapes and spatial relationships.

Children interpret the physical world with geometric ideas (e.g., shape, orientation, spatial relations) and describe it with corresponding vocabulary. They identify, name, and describe a variety of two- and three-dimensional shapes presented in a variety of ways (e.g., different sizes or orientations). They use basic shapes and spatial reasoning to model objects in their environment and to construct more complex shapes.

Learning Goals and Performance Indicators:

1) Understand that two and three-dimensional shapes can be classified on the basis of their attributes.

    a) *Identify, name, and describe two- and three-dimensional shapes including circles, triangles, rectangles, squares (as special rectangles), pyramids, rectangular prisms, cubes (as special rectangular prisms), cones, cylinders, and spheres.
    b) *Identify and describe shapes represented in the environment.
    c) *Use multiple ways to sort objects into groups by attribute (shape or size) and identify which attribute was used.

2) Understand relative positions of objects in space.

    a) *Recognize that when shapes are moved, they maintain the same classification.
    b) *Demonstrate relative positions in space (*over, under, above, below, on, beside, next to, and between*).

* *Locally assessed items.*

Kindergarten, Core Concept D:
Develop an understanding of measurable attributes and the informal process of measurement to compare and order objects.

Children use measurable attributes, such as length or weight, to solve problems by comparing and ordering objects. They compare the lengths of two objects both directly (by comparing them with each other) and indirectly (by comparing both with a third object), and they order several objects according to length. They experiment with and use rulers and scales as tools of measurement for length and weight.

Learning Goals and Performance Indicators:

1) Understand measurable attributes of objects through informal measurement of objects.

   a) *Select an appropriate tool for the attribute being measured (length and weight).
   b) *Compare and order objects indirectly or directly by using the measurable attributes of length and weight.

* *Locally assessed items.*

GRADE 1

Grade 1, Core Concept A:
Develop an understanding of whole number patterns and relationships, including grouping in tens and ones.

Children compare and order whole numbers to develop an understanding of and solve problems involving the relative sizes of these numbers. They think of whole numbers between 10 and 100 in groups of tens and ones (especially recognizing the numbers 11 to 19 as 1 group of ten and particular numbers of ones). They understand the sequential order of the counting numbers as well as their relative magnitudes and represent numbers on a number line.

Learning Goals and Performance Indicators:

1) Understand the relative magnitude of two-digit numbers.

    a) *Translate among a variety of representations for two-digit numbers, including objects, diagrams, number lines, words, and numerals.
    b) *Read, write, compare and order numbers through at least 100.

2) Understand the relationships among the values of the digits in two-digit numbers.

    a) *Recognize the place value (tens, ones) and value of each digit in a number (e.g., the 6 in 63 is 60 ones or 6 tens).
    b) *Translate among different numerical representations of a number (e.g., 63 tens is 6 tens and 3 ones or 5 tens and 13 ones or 4 tens and 23 ones).
    c) *Identify one more, one less, 10 more, and 10 less than a given number when the result is less than 100.

3) Understand growing patterns and make conjectures about the properties of odd and even numbers.

    a) *Count by 2s, 5s and 10s forwards and backwards, and connect these counting patterns with the operations of addition and subtraction.
    b) *Describe how simple growing patterns (increasing and decreasing) are generated**.**
    c) *Identify numbers missing from a well-defined counting sequence.
    d) *Make conjectures about the patterns and properties of even and odd numbers.

** Locally assessed items.*

Grade 1, Core Concept B:
Develop an understanding of addition and subtraction, apply a variety of strategies for basic addition/subtraction facts, and relate addition and subtraction as inverse operations.

Children develop strategies for adding and subtracting whole numbers on the basis of their earlier work with small numbers. They use a variety of models to develop an understanding of the meanings of addition and subtraction and strategies to solve such arithmetic problems. Children understand the connections between the patterns of counting up and counting back with the operations of addition and subtraction (e.g., adding two is the same as "counting on" two). They use properties of addition (commutativity and associativity) to add whole numbers, and they create and use increasingly sophisticated strategies based on these properties to solve addition and subtraction problems involving basic facts. They represent these mathematical situations with diagrams, words, and symbols. By comparing a variety of solution strategies, children relate addition and subtraction as inverse operations.

Learning Goals and Performance Indicators:

1) Understand the meanings of simple addition and subtraction (with sums and minuends to 20) along with related concepts of place value, properties of operations, and relationships between operations.

    a) *Make connections among a variety of representations, including objects, length-based models (e.g., lengths of connecting cubes), number lines, ten frames, diagrams, words, and number sentences, in order to explain addition and subtraction situations (combining, separating, comparing, and relating parts with wholes).
    b) *Compare solution strategies in order to relate addition and subtraction as inverse operations.
    c) *Explain and justify simple addition and subtraction strategies on the basis of properties of operations (identity, commutativity, associativity), place value, and/or the inverse relationship between addition and subtraction.

2) Understand and use a variety of strategies (involving concepts of place value, properties of operations, and relationships among operations) to compute simple addition and subtraction problems with sums and minuends to 20.

    a) *Interpret the equal sign as a relational symbol indicating "the same quantity as."
    b) *Use various forms of equations that involve addition and subtraction (e.g., 3 + 4 = __; 7 – __ = 3; 3 + 4 = __ + 2; __ = 5 – 2).
    c) *Apply and justify the use of strategies involving known facts, derived facts (i.e., doubling plus one, doubling minus one, making ten), counting on, and counting back to solve problems, including basic addition and subtraction facts with sums and minuends to 20.
    d) *Create and solve simple contextual problems involving addition and/or subtraction for a variety of mathematical situations (combining, separating, comparing, and relating parts with wholes).

** Locally assessed items.*

Grade 1, Core Concept C:
Develop an understanding of the relationships among geometric shapes

Children compose and decompose plane and solid figures (e.g., by putting two congruent isosceles triangles together to make a rhombus), thus building an understanding of part-whole relationships as well as the properties of the original and composite shapes. As they combine figures, they recognize them from different perspectives and orientations, describe their geometric attributes and properties, and determine how they are alike and different. Through these processes, they develop a background for measurement (e.g., area and volume) and initial understandings of such properties as congruence and symmetry.

Learning Goals and Performance Indicators:

1) Understand part-whole relationships and attributes of plane and solid figures.

   a) *Name, create, and sort geometric plane figures including circles, triangles, rectangles, squares (as special rectangles), rhombi, trapezoids, and hexagons.
   b) *Name and sort solid figures including pyramids, rectangular prisms, cubes (as special rectangular prisms), cones, cylinders, and spheres.
   c) *Compose (combine) and decompose (separate) two- and three-dimensional shapes to make other shapes.
   d) *Predict the number of congruent shapes that will cover a region without gaps, then test and verify predictions with models.
   e) *Recognize that when shapes are reflected (flipped), rotated (turned), or translated (slid), they maintain the same classification.
   f) *Identify two-dimensional shapes that have line (mirror) symmetry and draw line(s) of symmetry.

** Locally assessed items.*

GRADE 2

Grade 2, Core Concept A:
Develop an understanding of the base-ten numeration system and place-value concepts.

Children develop an understanding of the base-ten numeration system and place-value concepts (at least to 1000). Their understanding of base-ten numeration includes counting in units and multiples of hundreds, tens, and ones, as well as a grasp of number relationships, which they demonstrate in a variety of ways, including comparing and ordering numbers. They understand multi-digit numbers in terms of place value, recognizing that place-value notation is a shorthand for the sums of multiples of powers of 10 (e.g., 853 as 8 hundreds + 5 tens + 3 ones; 800 + 50 + 3).

Learning Goals and Performance Indicators:

1) Understand the relative magnitude of three-digit numbers.

   a) *Translate among a variety of representations of three-digit whole numbers, including base ten models, diagrams, words, standard and expanded forms.
   b) *Read, write, compare and order multi-digit numbers through 1000 by using terms and symbols for *equal to (=), greater than (>),* and *less than (<).*

2) Understand the relationships among the values of the digits in three-digit numbers.

   a) *Recognize the place value (hundreds, tens, ones) and value of each digit in a number (e.g., the 6 in 678 represents 6 hundreds, 60 tens, or 600 ones).
   b) *Translate among different numerical representations of a number (e.g., 678 is 6 hundreds, 7 tens, and 8 ones or 6 hundreds, 6 tens, and 18 ones, or 4 hundreds, 27 tens, and 6 ones).
   c) *Identify 10 more or 10 less and 100 more or 100 less than a given number when the result is less than 1000.

* *Locally assessed items.*

Grade 2, Core Concept B:
Develop quick recall with basic related addition/subtraction facts, demonstrate fluency with multi-digit addition and subtraction strategies, and explain why strategies work on the basis of place value and properties of operations.

Children use their understanding of addition to develop quick recall of basic addition facts and related subtraction facts. They solve arithmetic problems by applying their understanding of models of addition and subtraction (such as combining or separating sets or using number lines), relationships and properties of number (such as place value), and properties of addition. Children develop, discuss, and use efficient, accurate, and generalizable methods to add and subtract multi-digit whole numbers. They select and apply appropriate methods to estimate sums and differences or calculate them mentally, depending on the context and numbers involved. They develop fluency with computational procedures, including standard algorithms, for adding and subtracting whole numbers, understand why the procedures work (on the basis of place value and properties of operations), and use them to solve problems. They represent these mathematical relationships with diagrams, words, and symbols.

Learning Goals and Performance Indicators:

1) Understand the meanings of multi-digit addition and subtraction (with sums and minuends to 1000) along with related concepts of place value, properties of operations, and relationships between operations.

    a) *Make connections among representations (objects, diagrams, words, expressions and equations) of addition and/or subtraction situations (combining, separating, comparing, and relating parts with wholes).
    b) *Analyze a variety of strategies (including standard algorithms) for addition and subtraction in order to demonstrate their similarities and differences, and to draw conclusions about their efficiency, accuracy, and generalizability.
    c) *Explain and justify multi-digit addition and subtraction strategies on the basis of place value concepts, properties of operations (identity, commutativity, associativity) and/or the inverse relationship between addition and subtraction.

2) Understand and use a variety of strategies (involving concepts of place value, properties of operations, and relationships between operations) to compute multi-digit addition and subtraction problems (with sums and minuends to 1000) fluently.

    a) *Recognize and apply the meaning of relational signs (=, ☐, <, >) as distinct from operational signs (+, –) and use these symbols in a variety of equations and inequalities (e.g., 36 + 4 = ___, 32 = 71 – ___, 3 + 4 = ___ + 2, ___ > 5 – 2).
    b) *Demonstrate quick recall (through known and derived facts) of basic addition and subtraction facts with sums and minuends to 20.
    c) *Apply and justify the use of a variety of strategies, (including standard algorithms), to compute multi-digit addition and subtraction problems fluently (efficiently, accurately, and flexibly).
    d) *Estimate sums and differences and/or calculate them mentally, depending on the context and numbers involved.
    e) *Create and solve contextual problems for a variety of mathematical situations (combining, separating, comparing, and relating parts with wholes) which involve multi-digit numbers.

** Locally assessed items.*

Grade 2, Core Concept C:
Develop an understanding of linear measurement and facility in measuring lengths.
Children develop an understanding of the meaning and processes of measurement, including such underlying concepts as partitioning (the mental activity of slicing the length of an object into equal-sized units) and transitivity (e.g., if object A is longer than object B and object B is longer than object C, then object A is longer than object C). They understand linear measure as an iteration of units (repetition of a single unit with no gaps or overlaps) and use rulers and other measurement tools with that understanding. They understand the need for equal-length units, the use of standard units of measure (centimeter and inch), and the inverse relationship between the size of a unit and the number of units used in a particular measurement (i.e., children recognize that the smaller the unit, the more iterations they need to cover a given length).

Learning Goals and Performance Indicators:

1) Understand that the measurement process involves choosing a unit, comparing the unit to the object, and determining the number of units.

    a) *Use repetition of a single unit to measure the length of an object longer than the unit.
    b) *Describe the inverse relationship between the size of a unit and the number of units to measure an object.
    c) *Use direct comparison and measurement, along with concepts of the transitive property (a>b, b>c, so a>c) when comparing and ordering objects by the attribute of length.

2) Understand that linear measurements are approximations and that measuring tools provide the ability to make measurements with more precision.

    a) *Estimate length by using non-standard and standard units (customary and metric).
    b) *Choose and use non-standard and standard units (customary and metric) to measure length to the nearest whole unit.
    c) *Realize that when multiple identical units are used for measurement, they must match up with the beginning of the object being measured and there can be no gaps or overlaps between units.
    d) *Recognize each unit on a ruler as a linear distance (length), not a location on the tool.
    e) *Position the ruler so that the starting point (the zero point) of the ruler is at the beginning of the object being measured.

** Locally assessed items.*

Grade 2, Core Concept D:
Develop an initial understanding of data analysis by formulating questions and simple experiments, collecting, representing, analyzing, and interpreting data.

With teacher assistance, students pose and investigate questions that can be addressed by collecting categorical data about a small population. They conduct classroom censuses and use a variety of displays to represent and interpret data.

Learning Goals and Performance Indicators:

1) Understand how to collect, represent, and interpret categorical data in response to questions posed by the class.

   a) *Collect categorical data by conducting classroom censuses based on questions posed by the class.
   b) *Use one-to-one correspondence in constructing frequency tables (with tallies or numbers), bar graphs, and picture graphs in order to represent and interpret the data.
   c) *Make connections among the different representations of the same data and attend to the ideas of distribution and variation within the data set.
   d) *Understand that the mode is the category containing the most data points.
   e) *Compare responses (individual-to-individual and individual-to-group) and acknowledge that results may be different in another class or group.

** Locally assessed items.*

GRADE 3

Grade 3, Core Concept A:
Develop an initial understanding of multiplication and division, relate multiplication and division as inverse operations, and apply a variety of strategies for basic related multiplication/division facts.

Students understand the meanings of multiplication and division of whole numbers through the use of representations. They use properties of addition and multiplication to multiply whole numbers and apply increasingly sophisticated strategies based on these properties to solve multiplication and division problems involving basic facts. They represent these mathematical relationships with diagrams, words, and symbols. By comparing a variety of solution strategies, students relate multiplication and division as inverse operations. They build a foundation for later understanding of functional relationships by describing multiplicative relationships in context.

Learning Goals and Performance Indicators:

1) Understand meanings of multiplication and division (with products and dividends to 100) along with related concepts of place value, properties of operations, and relationships among operations.

    a) Make connections among representations of multiplication and division with equal-sized groups, arrays, area models, equal "jumps" on number lines, expressions and equations.
    b) Describe multiplicative relationships in context ("on 1 chair, there are 4 legs, on 2 chairs 8 legs, etc;" "the number of legs is 4 times the number of chairs").
    c) Use physical models and diagrams to demonstrate differences between partitive (sharing) and quotative (grouping) models of division.
    d) *Compare solution strategies in order to relate multiplication and division as inverse operations.
    e) Explain relationships among operations (between multiplication and addition, division and subtraction, multiplication and division).

2) Understand and use a variety of strategies (involving concepts of place value, properties of operations, and relationships among operations) to compute simple multiplication and division problems (including basic facts with products and dividends to 100).

    a) Recognize and apply the meaning of relational signs (=, ☐, <, >) as distinct from operational signs (+, –, ×, ÷) and use these symbols in a variety of equations and inequalities (e.g., $6 \times 4 =$ ___, $8 = 16 \div$ ___, $3 \times 4 =$ ___ $\times 2$, ___ $> 5 - 2$).
    b) *Apply and justify the use of identity, commutative, associative, and distributive (i.e., double and double again for fours-facts, fives-facts plus 1 set for six-facts) properties related to multiplication when solving simple multiplication and division problems.
    c) *Apply and justify the use of place value concepts and the inverse relationship of multiplication and division to solve simple multiplication and division problems.
    d) Interpret remainders appropriately in contextual problems.
    e) Create contextual problems for a variety of mathematical situations (addition, subtraction, simple multiplication and/or partitive and quotative division).
    f) Solve contextual problems for a variety of mathematical situations (addition, subtraction, simple multiplication and/or partitive and quotative division).

** Locally assessed items.*

Grade 3, Core Concept B:
Develop understanding of fractions and fraction equivalence by using area, length, and set models.

Students develop an understanding of the meanings and uses of fractions to represent parts of a whole, parts of a set, or points or distances on a number line. They understand that the size of a fractional part is relative to the size of the whole, and they use fractions to represent numbers that are equal to, less than, or greater than 1. They solve problems that involve comparing and ordering fractions by using models, benchmark fractions, or common numerators or denominators. They understand and use models, including the number line, to identify equivalent fractions.

Learning Goals and Performance Indicators:

1) Understand the meaning of fractions equal to, less than, or greater than 1.

    a) Translate among a variety of different representations of fractions, including objects, diagrams, number lines, words, and numerals.
    b) Use area models (e.g., geoboards, grid paper, dot paper, pattern blocks, rectangular regions), length models (e.g., rods, fractions strips, number lines), and set models (e.g., two-color counters, X and O drawings) to represent fraction concepts.
    c) Define regions, lines, and sets of objects as a whole and divide the whole into equal parts.
    d) Identify and apply the meaning of the denominator of a fraction as the number of equal parts of the unit whole and the numerator of a fraction as the number of equal parts being considered.
    e) Given a fraction, construct a whole.

2) Understand relationships among common fractions equal to, less than, or greater than 1.

    a) Compare and order fractions in a variety of ways such as using physical models of fractions, relating fractions to benchmarks (e.g., closer to 0, 1/2, or 1), and comparing fractions with common numerators or denominators.
    b) Use models, including the number line, to identify equivalent fractions.

* *Locally assessed items.*

Grade 3, Core Concept C:
Develop an understanding of properties and relationships in two-dimensional space.

Students describe, analyze, compare, and classify two-dimensional shapes by their sides and angles and connect these attributes to the definitions of shapes. Students investigate, describe, and reason about decomposing, combining, and transforming polygons to make other polygons. Through creating geometric patterns and by building, drawing, and analyzing two-dimensional shapes, students understand attributes and properties of two-dimensional space. They use these attributes and properties in solving problems, including applications involving congruence, symmetry, transformations, and perimeter.

Learning Goals and Performance Indicators:

1) Understand that two-dimensional shapes can be created, described, compared, and classified on the basis of their attributes and properties.

    a) Identify attributes for classifying triangles (e.g., two equal sides for the isosceles triangle, right angle for the right triangle).
    b) Identify attributes for classifying quadrilaterals (e.g., parallel sides for the parallelogram, right angles or perpendicular sides for the rectangle).
    c) Identify right angles in geometric figures and determine whether other angles are greater or less than a right angle.
    d) Create, describe, and/or extend geometric patterns involving changes in quantity, size, orientation and/or number of sides.
    e) *Predict the results of putting together and taking apart two-dimensional shapes, then test predictions with models.
    f) Create shapes meeting particular specifications such as number of sides, relative length of sides, or relationships between sides (e.g., triangle with no congruent sides, pentagon with two parallel sides).

2) Understand and apply concepts of perimeter and perimeter measurement.

    a) Recognize that any point on a ruler can be used as a starting point for measurement, as long as adjustments are made for non-zero starting points.
    b) *Use customary and metric units to estimate and measure the perimeter of real objects.
    c) Use customary and metric units to determine the perimeter of two-dimensional figures.
    d) Create a variety of rectangles with a given perimeter.
    e) Use perimeter concepts and knowledge of metric and customary measurement systems to solve problems involving rectangles.

3) Understand the meanings and uses of transformations.

    a) Use translations (slides), reflections (flips), and rotations (turns) to transform two-dimensional shapes.
    b) Determine whether two shapes are congruent by using a combination of translations, reflections, and/or rotations with models.
    c) Use transformations to determine whether a two-dimensional shape has line and/or rotational symmetry.

** Locally assessed items.*

Grade 3, Core Concept D:
Develop an understanding of the relationships within the measurement systems of time and money.

Students understand that measurement systems have structures which include related units and tools for measurement. Through hands-on experiences and real-world contexts, they explore concepts of money and time, move flexibly within each measurement structure, and solve related problems.

Learning Goals and Performance Indicators:

1) Understand the relationships within measurement systems of time and money and solve related problems.

    a) Describe relationships among adjacent units (e.g., seconds in a minute, hours in a day) for measurements of time, including seconds, minutes, hours, days, weeks and years.
    b) Tell time to the nearest five minutes.
    c) Determine elapsed time in hours.
    d) Make conversions among pennies, nickels, dimes, quarters, and dollars.
    e) Determine the value of a combination of coins and bills up to $10.00.
    f) Determine change from $10.00 and add and subtract money values to $10.00.

** Locally assessed items.*

GRADE 4

Grade 4, Core Concept A:
Develop quick recall of related multiplication/division facts, demonstrate fluency with whole number multiplication strategies, and explain why strategies work on the basis of place value and properties of operations.

Students use understandings of multiplication to develop quick recall of the basic multiplication facts and related division facts. They apply their understanding of models for multiplication, place value of numbers, and properties of operations (in particular, the distributive property) as they develop, discuss, and use efficient, accurate, and generalizable methods to multiply multi-digit whole numbers. They select appropriate methods and apply them accurately to estimate products or calculate them mentally, depending on the context and numbers involved. They develop fluency (accuracy, efficiency, and flexibility) with computational procedures, including the standard algorithm, for multiplying whole numbers. They understand why the procedures work (on the basis of place value and properties of operations), use them to solve problems. They represent these mathematical situations with diagrams, words, and symbols. In preparation for and as a result of working with larger numbers in multiplication, students also expand their concepts of place value.

Learning Goals and Performance Indicators:

1) Understand multi-digit multiplication (with products to 10,000) and related concepts of place value and properties of operations.

    a) Make connections among representations of multi-digit multiplication situations with objects, diagrams, words, expressions, and equations.
    b) Determine the place value (hundred thousands through ones) and the value of each digit in a number (e.g., the 6 in 678,000 represents 6 hundred thousands, 60 ten thousands or 600 thousands) and translate among these representations.
    c) Demonstrate the effects of place value when multiplying whole numbers by 10 or 100.
    d) *Analyze a variety of strategies (including standard algorithms) for multiplication in order to demonstrate their similarities and differences, and to draw conclusions about their efficiency, accuracy, and generalizability.
    e) *Explain and justify multi-digit multiplication strategies on the basis of place value and properties of operations (identity, zero, commutativity, associativity, and distributivity).

2) Understand and use a variety of strategies (involving related concepts of place value and properties of operations) to compute multiplication problems (with products to 10,000) fluently.

    a) *Demonstrate quick recall (through known and derived facts) of basic multiplication and division facts with products and dividends to 100.
    b) Apply and justify the use of a variety of strategies, including standard algorithms, to compute multi-digit multiplication problems fluently (efficiently, accurately, and flexibly).
    c) *Estimate products and/or calculate them mentally depending on the context and numbers involved.
    d) Create contextual problems for a variety of mathematical situations (addition, subtraction, multi-digit multiplication and/or simple partitive [sharing] and quotative [grouping] division).
    e) Solve contextual problems for a variety of mathematical situations (addition, subtraction, multi-digit multiplication and/or simple partitive [sharing] and quotative [grouping] division).

** Locally assessed items.*

Grade 4, Core Concept B:
Develop understanding of decimals, including the connections between fractions and decimals.

Students understand decimal notation as an extension of the base-ten system of writing whole numbers that is useful for representing more numbers, including numbers between 0 and 1, between 1 and 2, and so on. Students relate their understanding of fractions to reading and writing decimals greater than or less than 1, identifying equivalent decimals, comparing and ordering decimals, and estimating decimal or fractional amounts in problem solving. They connect equivalent fractions and decimals by comparing models to symbols and locating equivalent symbols on the number line.

Learning Goals and Performance Indicators:

1) Understand the meaning of decimals and the relationships among the values of their digits.

    a) *Recognize that the base-ten number system extends infinitely in two directions (from very small values to very large values).
    b) Translate among a variety of representations of hundredths and tenths, including base ten models, meter sticks, words, standard and expanded forms.
    c) Recognize and apply the concepts underlying place value (tenths, hundredths) by value of each digit in a decimal (e.g., the 7 in 0.78 represents 7 tenths or 70 hundredths) and translate among these representations.
    d) Identify a tenth more or a tenth less, and a hundredth more or a hundredth less, than a given number.

2) Understand relationships among whole numbers, commonly used fractions, and decimals.

    a) Model fractions (halves, fifths fourths, eighths, and tenths) on a 10 x 10 grid representing one unit in order to convert fractions to decimals.
    b) Rename whole numbers as fractions with different denominators (e.g., 5 = 5/1, 3 = 6/2, 1 = 7/7), with or without models.
    c) Relate fractions with denominators of tenths and hundredths to decimals of equivalent values.
    d) Identify equivalent fractions and decimals (less than one, equivalent to one, and greater than one), with and without models, including locations on a number line.
    e) Use a variety of methods to compare and order decimals and fractions.

** Locally assessed items.*

Grade 4, Core Concept C:
Develop an understanding of attributes of two-dimensional shapes by exploring angles, angle classifications, area, and area measurement.

Students establish benchmarks for estimating the degrees in angles and they classify angles by size. They recognize area as an attribute of two-dimensional regions. They learn that they can quantify area by finding the total number of same-sized units of area that cover the shape without gaps or overlaps. They understand that a plane-covering unit is needed to measure area and that a square which is 1 unit on a side is the standard unit for this measurement. Students connect area measure to the area model that they have used to represent multiplication, and they use this connection to justify the formula for the area of a rectangle.

Learning Goals and Performance Indicators:

1) Understand concepts of angle and angle measurement.

   a) *Describe and interpret angles in various contexts, including static angles in geometric figures and dynamic angles involving motion (e.g., hands of a clock, the opening of a door).
   b) Recognize that the length of an angle's sides does not affect the measure of an angle.
   c) Use physical models such as pattern blocks and tangrams to establish benchmarks (45°, 90°, 120°, 180°, and 270°) angle measures.
   d) Use benchmarks to estimate angle measures and to draw angles.
   e) Classify angles as right, acute, or obtuse.

2) Understand the concepts of area and area measurement as applied to rectangles.

   a) Find the area of two-dimensional figures by using physical models (e.g., square units to cover a shape with no gaps or overlaps or a transparent grid placed over a shape) to count the total number of units.
   b) Derive and use the area formula for a rectangle and connect it with the area model for multiplication.
   c) Create a variety of rectangles with a given area.
   d) Use area concepts and knowledge of metric and customary measurement systems to solve problems involving rectangles.
   e) Distinguish between appropriate units for linear and area measurement situations.

* *Locally assessed items.*

Grade 4, Core Concept D:
Develop an understanding of data analysis by formulating questions, collecting or using available data, analyzing tools of data representation, and using measures of center and spread to interpret data.

Students formulate and investigate questions that can be addressed with numerical data. They conduct simple experiments and use a variety of displays to represent and interpret data. Students attend to the distribution of data by describing its center and spread, and recognize limitations of the scope of inference beyond the experiment.

Learning Goals and Performance Indicators:

1) Understand how to formulate and answer questions that can be addressed by conducting simple experiments and collecting numerical data.

    a) *Formulate questions that can be addressed with data.
    b) Collect or use available numerical data in order to draw conclusions and answer questions.

2) Understand and apply tools for exploring distributions (including measures of center and spread), the appropriateness of data displays, and limitations of inference.

    a) Represent distributions of data by using a variety of displays, including tables, bar graphs, line graphs, line plots, and stem-and-leaf plots, and discuss the appropriateness of each type of display.
    b) Describe the distribution of data in terms of its center (mode and median) and spread (the range of responses).
    c) Compare related data sets on the basis of measures of center (mode and median) and spread (the range of responses).
    d) Recognize limitations in the scope of inference beyond the experiment.

** Locally assessed items.*

GRADE 5

Grade 5, Core Concept A:
Develop an understanding of and fluency with whole number division strategies and explain why strategies work on the basis of place value and properties of operations.

Students apply their understanding of models for division, place value, properties, and the relationship of division to multiplication as they develop, discuss, and use efficient, accurate, and generalizable procedures to find quotients involving multidigit dividends. They select appropriate methods and apply them accurately to estimate quotients or calculate them mentally, depending on the context and numbers involved. They develop fluency with computational procedures, including the standard algorithm, for dividing whole numbers, understand why the procedures work (on the basis of place value and properties of operations), and use them to solve problems. They consider the context in which a problem is situated to select the most useful form of the quotient for the solution, and they interpret it appropriately. By comparing a variety of solution strategies, children deepen their understanding of multiplication and division as inverse operations. They represent these mathematical situations with diagrams, words, and symbols.

Learning Goals and Performance Indicators:

1) Understand multi-digit division (with divisors to 100 and dividends to 10,000) and related concepts of place value, properties of operations, and relationships among operations.

    a) Make connections among representations of multi-digit division situations with objects, diagrams, words, expressions, and equations.
    b) Demonstrate the effect of place value when dividing by 10 or 100.
    c) *Analyze a variety of strategies (including standard algorithms) for division in order to demonstrate their similarities and differences, and to draw conclusions about their efficiency, accuracy, and generalizability.
    d) *Explain and justify multi-digit division strategies on the basis of place value, relationships among operations, and properties of operations (identity, distributive).

2) Understand and use a variety of strategies (involving concepts of place value, properties of operations, and relationships among operations) to compute division problems (with divisors to 100 and dividends to 10,000) fluently.

    a) Apply and justify the use of a variety of strategies, including standard algorithms, to compute division problems with two-digit divisors fluently (efficiently, accurately, and flexibly).
    b) Express remainders as whole numbers or fractions and interpret remainders appropriately in contextual problems.
    c) *Estimate products and quotients and/or calculate them mentally depending on the context and numbers involved.
    d) Create contextual problems involving any combinations of whole number operations.
    e) Solve contextual problems involving any combinations of whole number operations.

** Locally assessed items.*

Grade 5, Core Concept B:
Develop an understanding of and fluency with addition and subtraction strategies for fractions (including mixed numbers) and decimals, and explain why strategies work on the basis of place value and properties of operations.
Students apply their understanding of fractions and fraction models to represent the addition and subtraction of fractions with unlike denominators as equivalent calculations with like denominators. They apply their understanding of decimal models, place value, and properties to add and subtract decimals. They develop fluency with standard procedures for adding and subtraction fractions and decimals and make reasonable estimates of fraction and decimal sums and differences. Students add and subtract fractions and decimals to solve problems, including problems involving measurement. They represent these mathematical situations with diagrams, words, and symbols.

Learning Goals and Performance Indicators:

1) Understand and apply strategies to add and subtract fractions (including mixed numbers with like and unlike denominators).

    a) Make connections among representations of fraction addition and subtraction situations with objects, diagrams, words, expressions and equations.
    b) Apply knowledge of common multiples, common factors, prime and composite numbers to support the addition and subtraction of fractions.
    c) *Analyze a variety of strategies (including standard procedures) for addition and subtraction of fractions in order to demonstrate their similarities and differences, and to draw conclusions about their efficiency, accuracy, and generalizability.
    d) Apply and justify a variety of strategies, including standard procedures, to add and subtract fractions, including mixed numbers, fluently (efficiently, accurately, and flexibly).
    e) *Estimate fraction sums and differences and/or calculate them mentally, depending on the context and numbers involved.
    f) Create contextual problems for a variety of mathematical situations (combining, separating, comparing, and relating parts with wholes) involving addition and subtraction of fractions.
    g) Solve contextual problems for a variety of mathematical situations (combining, separating, comparing, and relating parts with wholes) involving addition and subtraction of fractions.

2) Understand and apply strategies to add and subtract decimals.

    a) Make connections among representations of decimal addition and subtraction situations with objects, diagrams, words, expressions and equations.
    b) *Relate the addition and subtraction of decimals to the addition and subtraction of fractions.
    c) *Analyze a variety of strategies (including standard procedures) for addition and subtraction of decimals in order to demonstrate their similarities and differences, and to draw conclusions about their efficiency, accuracy, and generalizability.
    d) Apply and justify a variety of strategies, including standard procedures, to add and subtract decimals fluently (efficiently, accurately, and flexibly).
    e) *Estimate decimal sums and differences and/or calculate them mentally, depending on the context and numbers involved.
    f) Create contextual problems for a variety of mathematical situations (combining, separating, comparing, and relating parts with wholes) involving addition and subtraction of decimals.
    g) Solve contextual problems for a variety of mathematical situations (combining, separating, comparing, and relating parts with wholes) involving addition and subtraction of decimals.

** Locally assessed items.*

Grade 5, Core Concept C:
Develop an understanding of the properties of three-dimensional shapes, including volume and surface area.

Students relate two-dimensional shapes to three-dimensional shapes and analyze properties of polyhedral solids, describing them by the number of edges, faces, or vertices as well as the types of faces. Students recognize volume as an attribute of three-dimensional space. They understand that they can quantify volume by finding the total number of same-sized units of volume that they need to fill the space without gaps or overlaps. They understand that a space-filling unit is needed to measure volume and that a cube which is 1 unit on an edge is the standard unit for this measurement. They select appropriate units, strategies, and tools for solving problems that involve estimating or measuring volume. They decompose three-dimensional shapes and find surface areas and volumes of prisms. As they work with surface area, they find and justify relationships among the formulas for the areas of different polygons. They measure necessary attributes of shapes to use area formulas to solve problems.

Learning Goals and Performance Indicators:

1) Understand the properties of three-dimensional shapes.

    a) Determine the number of edges, faces, and vertices of a given polyhedron.
    b) Identify the types of faces of a given polyhedron.
    c) *Compose larger polyhedra from smaller ones; decompose larger polyhedra into smaller ones.
    d) Analyze and compare three-dimensional shapes on the basis of their edges, faces, and vertices.
    e) Relate triangular and rectangular pyramids and rectangular prisms to the two-dimensional shapes (nets) from which they were created.

2) Understand how areas of triangles are related to rectangular area and connect these ideas to concepts of surface area (nets) for rectangular prisms and rectangular or triangular pyramids.

    a) *Derive the area formula for triangles on the basis of their relationships with rectangles.
    b) Use area concepts and knowledge of metric and customary measurement systems to solve problems involving triangles.
    c) Use area concepts to solve problems involving surface areas (areas of nets) of rectangular or triangular pyramids and rectangular prisms.

3) Understand the concepts of volume and volume measurement.

    a) *Find the volume of rectangular prisms by using cubic units to fill them, with no gaps or overlaps, then counting the total number of units.
    b) *Identify, organize, and use the underlying structure of cubes filling a rectangular prism (a series of layers) to find the volume of rectangular prisms.
    c) *Solve problems that involve estimating or physically measuring the volume of rectangular prisms.
    d) Distinguish between appropriate units for linear, area, and volume measurement situations.

** Locally assessed items.*

Grade 5, Core Concept D:
Develop an understanding of probability through the contexts of simple experiments and their outcomes.

Students begin to describe events as likely by using such words as *certain, equally likely,* and *impossible*. They learn that probability is a measurement of the likelihood of events and quantify the probability of an event as a number between 0 and 1. Moreover, they learn that sample space is the set of all possible outcomes and that the sum of the probabilities of all sample space outcomes is 1. They begin to quantify likelihood by conducting experiments that have only a few outcomes. Through these experiences, students encounter the idea that although they cannot determine an individual outcome, they can predict the frequency of various outcomes.

Learning Goals and Performance Indicators

1) Understand that probability is a measurement of the likelihood of events and that the probability of an event is based on the set of all possible events called the sample space.

   a) List all possible outcomes for simple experiments (e.g., with number cubes and spinners).
   b) Recognize whether an outcome of simple event in an experiment or simulation is *impossible, unlikely, possible, likely,* or *certain*, and whether two or more events are *equally likely*.
   c) Represent the probability of an event, which ranges from 0 (*impossible*) to 1 (*certain*), with a fraction, decimal, or percent.
   d) Recognize that the sum of the probabilities of all events comprising the sample space is 1, and that if *P* is the probability of an event, then *1-P* is the probability of the event not occurring (the complement of the event).
   e) Predict the likelihood of an outcome prior to an experiment and compare the predicted probability with the experimental results.

* *Locally assessed items.*

GRADE 6

Grade 6, Core Concept A:
Develop an understanding of and fluency with multiplication and division of rational numbers and extend this understanding to ratio and rates.

Students understand the relationships among and establish fluency with equivalent representations of rational numbers including fractions, decimal, and percents. They use the meaning of fractions, multiplication and division, and the inverse relationship between multiplication and division to make sense of procedures for multiplying and dividing fractions. They use the relationships between decimals and fractions to explain the procedures for multiplying and dividing decimals. Students use common procedures to multiply and divide fractions and decimals efficiently and accurately, and to solve problems. Students develop the ability to use ratios and rates to represent and reason about comparisons and to solve problems. They understand that exponents represent repeated multiplication and understand the relationship between perfect squares and their roots.

Learning Goals and Performance Indicators:

1) Become fluent in finding equivalent representations of fractions, decimals, and percents; and in estimating, multiplying and dividing with nonnegative rational numbers, in fraction and decimal forms, as appropriate to given problem situations.

    a) Identify, compare, and order equivalent representations of nonnegative rational numbers (fractions, decimals, and percents) and translate fluently among these representations.
    b) *Select and apply an appropriate fraction, percent, or decimal representation for nonnegative numbers in a given context.
    c) Compute fluently, using a variety of strategies, including the standard algorithm, in problem situations involving multiplication and division of nonnegative rational numbers in fraction and decimal form.
    d) Make reasonable estimates for the products and quotients of nonnegative rational numbers.
    e) *Compare, justify, and apply strategies (including the standard algorithm) for multiplication and division of nonnegative rational numbers.

2) Identify and represent ratios and rates as comparisons, and reason to find equivalent ratios in problem situations.

    a) Identify and apply one or more ratios that represent a given comparison and express the ratio using appropriate notation.
    b) *Justify why two different ratios for a given comparison are equivalent in problem situations.
    c) Represent and model ratios associated with whole-number percents that are less than or equal to 100%.

3) Develop number sense related to natural number exponents.

    a) *Use exponents to represent repeated multiplication and calculate the value of expressions represented with exponential notation
    b) Identify and apply roots of perfect squares and perfect cubes.
    c) *Identify the two consecutive whole numbers between which the square root of a non-perfect square whole number is located.
    d) Apply the order of operations to simplify numerical expressions with exponents.

* *Locally assessed items.*

Grade 6, Core Concept B:
Explore, represent, and generalize patterns.

Students explore, represent, analyze and generalize numeric and geometric patterns with manipulatives, tables, graphs, and words. Students understand that variables represent an unknown quantity that has the potential to change and use them in writing expressions and equations to represent a given situation. Students evaluate expressions and understand that expressions in different forms can be equivalent. They use properties (distributive, associative, commutative) to rewrite an expression to represent a quantity in a different way (e.g., to make it more compact or to feature different information).

Learning Goals and Performance Indicators:

1) Represent, analyze, and generalize a variety of patterns with tables, graphs, and words.

    a) Describe geometric or numeric patterns, represented with tables or graphs, in words.
    b) Recognize, generate, and translate among representations of the same pattern (e.g., given a table of values, students will describe the pattern in words and create a graph of the pattern).

2) Write mathematical expressions and equations that correspond to a given situation, evaluate expressions, and use expressions and formulas to solve problems.

    a) Create, evaluate, and simplify expressions to represent contextual situations.
    b) *Solve simple equations generated from representing mathematics in context using informal strategies.
    c) Use properties (associative, commutative, distributives) to rewrite expressions.

* *Locally assessed items.*

Grade 6, Core Concept C:
Analyze properties of and measure two-dimensional shapes.

Students explore, identify, classify, and represent two-dimensional figures and their properties including angle measures, parallelism, perpendicularity, and symmetry. Students develop and apply formulas for measurements in the plane, especially those involving the perimeter/circumference and area of quadrilaterals and circles, and composite figures made from these shapes.

Learning Goals and Performance Indicators:

1) Represent, identify, and classify geometric figures from written or verbal descriptions, measurements, and properties using sketches, figures represented on the coordinate plane, grids, or models.

    a) Identify and use properties (including parallelism, perpendicularity, and symmetry) to classify two-dimensional figures.
    b) Draw or create two-dimensional figures or models with specified measures and properties including the use of first quadrant coordinates to construct geometric shapes.
    c) Measure angles using measurement tools and apply the basic properties associated with complementary and supplementary angles.
    d) Identify, state, and apply the Angle-Sum properties for triangles and other polygons.

2) Develop and apply formulas for perimeter/circumference and area of quadrilaterals, circles, and composite figures made from these shapes.

    a) *Describe the relationship between the circumference and diameter of a circle, $d\pi = C$, and apply it to develop convincing arguments about the validity of formulas such as $C = 2\pi r$ and $A = \pi r^2$.
    b) *Understand the relationships between the areas of various quadrilaterals and how these relationships are expressed in the formulas for the figures.
    c) Determine the circumference and area of circles.
    d) Determine the area of parallelograms and trapezoids.
    e) Find the perimeter and area of a composite polygonal figure by dividing it into known shapes.

* *Locally assessed items.*

Grade 6, Core Concept D:
Design and conduct sample surveys, explore random selection, and compare data distributions.

Students formulate and answer questions by collecting and analyzing univariate numerical data, exploring methods of random selection, and recognizing the distinction among a population, a census, and a sample. They summarize and compare two or more distributions using a variety of displays and numerical summaries and describe differences between them with respect to center, spread, and shape, and recognize limitations in the scope of inference beyond the experiment.

Learning Goals/Performance Indicators:

1) Formulate questions, identify the numerical attributes on which to collect data, decide how to measure the attribute, determine a data collection process; compare two or more distributions using displays and numerical summaries and describe differences between them with respect to center, spread, and shape, and recognize limitations in the scope of inference beyond the experiment.

    a) *Design and conduct sample surveys.
    b) Distinguish between a *population* and *sample*, and identify methods of sample selection.
    c) Summarize and compare distributions using numerical summaries and data displays, including relative frequency tables, box plots, circle graphs (pie charts), back-to-back stem-and-leaf plots, double bar graphs, and histograms.
    d) Quantify measures of center (mean, median, and mode), interpret the meaning of these measures in context, explain the influences of outliers on each measure, and justify which statistic is more appropriate for summarizing a given data set.
    e) Distinguish between interpretations of the mean as the "fair share" value for data and as the "balancing point" of the corresponding data distribution.

* *Locally assessed items.*

GRADE 7

Grade 7, Core Concept A:
Develop an understanding of and fluency with integers and their operations.

Students extend understandings of addition, subtraction, multiplication, and division, together with their properties to the entire set of integers. By applying properties of arithmetic and considering negative integers in everyday contexts, students explain why the rules for adding, subtracting, multiplying, and dividing with negative integers make sense. Students develop fluency in computing with integers and reason about and solve problems involving integers.

Learning Goals and Performance Indicators:

1) Represents, order, and compare integers.

   a) Represent different contexts using integers.
   b) Order and compare integers; locate integers on a number line; and recognize the absolute value as an integer's distance from zero on a number line.
   c) Locate and plot points whose coordinates are both integers on the four quadrant coordinate plane.

2) Model operations, compute fluently and solve problems with integers.

   a) *Model the addition, subtraction, multiplication, and division of integers using a variety of representations and describe the relationships among these operations.
   b) Compute fluently with integers in problem situations, applying order of operations.
   c) Estimate and judge the reasonableness of results involving integer operations.
   d) Identify, utilize, and justify properties (closure, associative, commutative, identity, inverse, zero) in computing with integers.

** Locally assessed items.*

Grade 7, Core Concept B:
Develop and apply an understanding of proportionality.

Students extend their work with ratios to develop an understanding of proportionality that they apply to solve single and multistep problems in numerous contexts. They use ratio and proportionality to solve a wide variety of percent problems, including problems involving discounts, interest, taxes, tips, and percent increase or decrease. They also solve problems about similar objects by using scale factors that relate corresponding lengths of the objects or by using the fact that relationships of lengths within an object are preserved in similar objects.

Learning Goals and Performance Indicators:

1) Develop computational fluency in working with ratios, percents, and proportional situations and apply this fluency to estimate the solution to and solve a variety of contextual problems.

    a) Use proportionality to model and solve contextual problems, including percent applications and measurement conversions, using a variety of strategies.
    b) Estimate to find solutions to percent problems.

2) Identify, describe, and apply similarity relationships to find measures of corresponding parts in similar figures and apply scales to measurements in drawings and maps.

    a) Given similar two-dimensional figures, identify the scale factor and describe the relationships between the scale factor and measurements of corresponding part (angles, side lengths, perimeters, areas).
    b) Determine if two figures are similar and justify the conclusion by examining corresponding side lengths, angles, perimeters, and area.
    c) Interpret and solve scaling problems involving various mathematical contexts (e.g. indirect measurement, scale models).

* *Locally assessed items.*

Grade 7, Core Concept C:
Develop and apply an understanding of transformations.

Students investigate the effects of transformations on geometric figures. They represent translations, reflections, rotations, and simple compositions of these transformations using sketches and coordinates, and explain the effects of these transformations. Students recognize and identify corresponding parts of the pre-image and image and recognize that these figures are similar. Students understand that symmetry results from transformations and identify lines of symmetry and angles of rotation.

Learning Goals and Performance Indicators:

1) Apply knowledge of transformations in relating objects and in their effects on figures.

   a) Locate the images of figures produced by transformations including translations, reflections about a vertical or horizontal line, rotations about the origin, and simple composition of these transformations.
   b) *Describes the effects of transformations including translations, reflections about a vertical or horizontal line, rotations about the origin, and simple composition of these transformations.
   c) Determine the coordinates of the image of a figure produced after a translation, a reflection about a vertical or horizontal line, or a rotation of a multiple of 90° about the origin of the coordinate plane
   d) Identify the corresponding parts (segments, angles, vertices) of the pre-image and image of a figure and identify the transformation that has occurred.
   e) Draw lines of symmetry and identify the angle of rotation in designs with rotational symmetry.

** Locally assessed items.*

Grade 7, Core Concept D:
Identify, represent and solve linear patterns and relationships.

Students identify linear patterns for mathematical contexts and represent linear expressions in words, tables, symbols, and graphs. Students write and solve one-step equations that represent linear patterns and contextual situations. Students make strategic choices of procedures to solve linear equations in one variable and implement them efficiently. They understand that when they use the properties of equality to express an equation in a new way, solutions that they obtain for the new equation also solve the original equation.

Learning Goals and Performance Indicators:

1) Create and evaluate simple linear expressions to represent linear patterns and develops graphs and tables to represent these expressions.

    a) Represent linear patterns generated by mathematical contexts with expressions using both explicit and recursive (Next, Now) notation, and evaluates these expressions for nonnegative rational numbers.
    b) Generate and graph a set of ordered pairs representing a given linear expression.
    c) Write rules in words and in symbols for situations modeled by the forms ax and $x \pm b$, where a and b are nonnegative rational numbers.
    d) Extend linear patterns drawn from mathematical contexts, sequences, tables, and graphs with verbal or symbolic rules of the form $ax \pm b$, where a and b are nonnegative rational numbers.
    e) Justify whether a relationship represented in a table, graph, or expression is linear or nonlinear.

2) Use linear patterns to create simple linear equations, and solve these equations.

    a) Use linear patterns drawn from contextual situations to create equations and solve problems.
    b) Evaluate nonnegative rational numbers as possible solutions to linear equations numerically and graphically, with and without technology.
    c) Write and solve linear equations of the form $ax = b$ and $x \pm b = c$, where a, b, and c and the solution are nonnegative rational numbers.

* *Locally assessed items.*

Grade 7, Core Concept E:
Estimate probabilities in experiments, comparing experimental and theoretical probabilities.

Students conduct one-stage experiments, used experimental data to estimate probabilities, and, where possible, compare experimental and theoretical probabilities and examine experimental probability in the long run. Students recognize that repetitions of an experiment may result in different outcomes, describe the variation in outcomes produced in an experiment. Students recognize that small samples are often not representative of the population from which they are drawn, and with the collection of more data the experimental probability of a particular outcome approaches the theoretical probability.

Learning Goals and Performance Indicators:

1) Conduct one-stage experiments to estimate the likelihood of a simple event, compare the experimental probability with an easily identifiable theoretical probability, describe and compare the likelihood of events.

   a) Determine the sample space for a given one-stage experiment; using lists, tables, and tree diagrams to represent all possible outcomes.
   b) Determine the theoretical probability of an event and its *complement* given a sample space.
   c) *Recognize that with the collection of more data, the experimental probability of a particular outcome approaches the theoretical probability and understand that although probability cannot determine an individual outcome, it can be used to predict the frequency of an outcome.
   d) Use experimental data to estimate the probability of an event when the theoretical probability is unknown.

** Locally assessed items.*

GRADE 8

Grade 8, Core Concept A:
Represent and solve linear relationships and inequalities.

Students develop and apply the connections between rate of change and linear relationships. Students create and solve linear equations using tabular and graphical displays, verbal representations of problems, and symbolic manipulation. Students interpret inequalities involving one variable. Students translate among verbal, tabular, graphical, and algebraic representations of functions (recognizing that tabular and graphical representations are usually only partial representations).

Learning Goals and Performance Indicators:

1) Interpret rate of change in contextual settings and recognize the constant rate of change associated with linear relationships.

    a) Recognize and demonstrate that the rate of change in linear settings is constant, and graphically describe the proportional relationship embedded in this rate of change and represented in the slope of the related line.
    b) Interpret, describe, and use rates of change to model contextual situations, noting the different patterns resulting from linear and exponential patterns.
    c) Identify and translate among equivalent representations (tables, graphs, equations) of linear relationships to solve problems.

2) Create one- and two-step linear equations to represent mathematical contexts and solve such equations using tables, coordinate graphs, and symbolic manipulation.

    a) Represent linear mathematical contexts using a linear equation of the form $ax + b = c$, where a, b, and c are rational numbers expressed as fractions, decimals, or integers.
    b) Solve linear equations with rational-number coefficients using mental, graphical, and symbolic methods, with and without technology.
    c) *Connect the graphical, tabular, and symbolic representations of the unique solution of a given linear equation.

3) Represent and interpret inequalities in one variable geometrically and symbolically.

    a) *Represent solutions to inequalities such as $x > a$ and $a \square x \square b$ on a number line.
    b) Write an inequality to represent an interval or ray, with or without endpoints, shown on the number line.

* *Locally assessed items.*

Grade 8, Core Concept B:
Develop understanding of real numbers, roots, and exponents.

Students develop fluency in estimating and computing with rational numbers including positive and negative fractions and decimals. They view the real number system as consisting of the rational and irrational numbers. They use positive- and negative-integer exponents and scientific notation to represent, order, and compare numbers. Students develop fluency in estimating real number operations and use properties of operations in solving problems.

Learning Goals and Performance Indicators:

1) Compute fluently to solve problems involving rational numbers written in both fraction and decimal form.

    a) Compute fluently with all rational numbers.
    b) Represent and solve problems involving rational numbers, and judge the reasonableness of the solutions.
    c) Read, write, compare and order numbers represented in scientific notation using positive- and negative-integer exponents for powers of 10, and interpret applications of scientific notation.

2) Distinguish between rational and irrational numbers, and use estimation to solve problems involving real numbers.

    a) Determine if a number is rational or irrational.
    b) Estimate solutions for problem situations involving operations and approximations with real numbers and judge the reasonableness of the results.

* *Locally assessed items.*

Grade 8, Core Concept C:
Understand and apply the Pythagorean Theorem and measure three-dimensional figures

Students explore three-dimensional figures to develop an understanding of the formulas for surface area and volume. They create and use isometric drawings, mat plans, and perspective drawings to represent three-dimensional figures. They investigate how surface area and volume are affected when one or more dimensions of a figure are changed, leading to the understanding of how scale factors impact surface area and volume. They apply these formulas to measure three-dimensional figures. Students develop the Pythagorean Theorem by investigating right triangles, their measures and related areas. They apply the Pythagorean Theorem to solve measurement problems.

Learning Goals and Performance Indicators:

1) Develop and apply the Pythagorean Theorem to solve for the lengths of sides in rights triangles and related measurement problems.

    a) *Justify the Pythagorean Theorem using a variety of methods.
    b) Apply the Pythagorean Theorem to solve measurement problems.

2) Apply the concepts of surface area and volume to measure and find formulas for measures of three-dimensional figures.

    a) *Describe the relationships between the measurements of three-dimensional figures and the measures of related two-dimensional figures. (For example, the volume of a prism can be found by multiplying the base area by the height.)
    b) Create and identify isometric drawings, from mat plans and models, from various perspectives.
    c) *Create and identify perspective drawings (e.g., front view, right view) from mat plans and models.
    d) Determine the surface areas and volumes of right prisms and right cylinders.
    e) Formulate and apply general statements relating whole number or unit fraction scale changes in the linear dimensions of a figure to the resulting changes in surface area and volume of the resulting figure.

** Locally assessed items.*

Grade 8, Core Concept D:
Organize and summarize bivariate data, represent and quantify associations between two variables using simple models.

Students formulate questions involving two attributes, design experiments, collect, organize, and summarize bivariate data using numerical and graphical displays to represent the data. Students quantify the strength of association between two variables, develop simple models for association, and understand basic interpretations of measures of association.

Learning Goals/Performance Indicators:

1) Formulate questions involving two attributes, design experiments, collect, organize, and summarize bivariate data using numerical and graphical displays to represent the data.

    a) *Design experiments and collect bivariate data to answer a question, classifying each attribute as a categorical or numerical variable.
    b) Identify, describe, and construct appropriate displays for bivariate data: *two-way tables* for two categorical variables; parallel *box plots* or back-to-back stem-and-leaf plots for one numerical and one categorical variable; and *scatterplots* for two numerical variables.
    c) Describe the distribution for each attribute separately using appropriate graphs, including stem-and-leaf plots, histograms, and box plots, and summary statistics, including five-number summaries and the *Inter-Quartile Range (IQR)*.

2) Quantify the strength of association between two variables using a variety of tools, develop simple models for association between two numerical variables, and understand basic interpretations of measures of association.

    a) Quantify the strength of association between two variables and develop simple models for association between two numerical variables including contingency tables (to display bivariate categorical data), the Quadrant Count Ratio (QCR) [Percentage of data in Quadrants I & III) and simple lines (e.g., median-median line) for modeling association between two numerical variables.
    a) Describe the relationship between the two variables, the effects of outliers on the observed relationship, and distinguish between an "association" and "cause and effect."

* *Locally assessed items.*

Algebra I, Core Concept A:
Interpret, model, justify and compare linear and nonlinear relationships.

Students identify situations with a constant rate of change that can be represented by linear functions, model these situations using algebraic, graphical, tabular, and verbal representations and translate among them, interpret the models, and solve contextual problems. They compare (similarities and differences) linear and nonlinear relationships using rates of change and graphical representations.

Learning Goals and Performance Indicators:

1) Identify functions based on their graphical behavior and rates of change, and describe functions using appropriate notation and terminology.

    a) Determine and justify whether a relationship is a function by using a graph or a verbal description of the relationship.
    b) Compare and justify linear or nonlinear relationships based on rate of change, verbal description, table of values, graphical representation, or symbolic form.
    c) Describe characteristics of piecewise-linear functions, including absolute value, and situations in which they arise.

2) Use linear functions to interpret, model, and solve situations having a constant rate of change. Use numeric patterns to generate symbolic patterns.

    a) Generalize linear patterns or arithmetic sequences using verbal rules and symbolic expressions such as kx and $ax + b$ in representing proportional or more-general linear relationships, respectively.
    b) Analyze a contextual situation; determine whether the situation can be described by a linear model, and if so, determine the constant rate of change and develop and interpret a linear function to model that situation.

* *Locally assessed items.*

Algebra 1, Core Concept B:
Interpret, represent and solve linear equations and inequalities.
Students distinguish among different uses of variables and find equivalent expressions and equations. They construct, represent, solve, and interpret solutions of linear equations, linear inequalities, and systems of linear equations for contextual situations. They determine the reasonableness of the solution(s) and extend their understanding of the real number system through solving problems in algebraic situations.

Learning Goals and Performance Indicators:

1) Represent linear patterns using expressions, equations, functions, and inequalities and interpret the meanings of these representations, recognizing which are equivalent and which are not.
   a) Describe the meaning of symbolic expressions and functions of the form $ax + b$ and f(x)=ax + b in words, and interpret the changes resulting from different values of the parameters a and b.
   b) Develop and justify equivalent algebraic expressions, equations, and inequalities using the properties of equality and inequality, as well as the commutative, associative, inverse, identity, and distributive properties.
   c) Identify and translate among equivalent representations of linear expressions, equations, inequalities, and systems of equations, using verbal, tabular, graphical, and symbolic representations.
   d) Write, interpret, and translate among equivalent forms of linear equations and functions, including slope-intercept, point-slope, intercept, and general (standard) forms, recognizing that equivalent forms for a linear relationship reveal more or less information about a given situation.
   e) Judge whether a scatterplot appears to show a linear trend, and if it does, draw a trend line and write an equation for that line; use the equation to make predictions; and interpret the slope of the line in context.

2) Distinguish among the different uses of variables, parameters, constants, and equations.
   a) Identify and distinguish among parameters and the independent and dependent variables.
   b) Describe and distinguish among the types of equations that can be constructed by equating linear expressions, including identities (e.g., $x + x = 2x$); equations for which there is no solution (e.g., $x + 1 = x + 2$); formulas (e.g., $C = \pi d$); equations where the solution is unique (e.g., $2x + 3 = 16$); and equations relating two variables (e.g., $y = 3x + 7$).
   c) Identify terms in an arithmetic (linear) sequence using verbal rules or symbolic expressions (explicit and recursive).

3) Constructs, solve, and interpret solutions of linear equations, linear inequalities, and systems of linear equations (limited to two equations with two unknowns) in contextual situations.
   a) Construct a linear equation or linear inequality to model a contextual situation, using a variety of methods and representations.
   b) Analyze and explain the reasoning used to solve linear equations and linear inequalities using symbolic methods, graphs, tables, and technology.
   c) *Construct a system of linear equations modeling a contextual situation, using a variety of methods and representations.
   d) *Analyze and explain the reasoning used to solve a system of linear equations using graphs, tables, symbolic methods, and technology, and describe the nature of the solutions (no solution, one solution, infinitely many solutions).

* *Locally assessed items.*

Algebra 1, Core Concept C:
Model, interpret, represent and solve nonlinear functions.

Students identify and classify non-linear relationships. They represent simple contextual phenomena using exponential and quadratic functions and solve equations involving these functions with a variety of techniques. They determine the reasonableness of the solution(s) and extend their understanding of the real number system through solving problems in algebraic situations.

Learning Goals and Performance Expectations:

1) Identify certain nonlinear relationships and classify them as exponential relationships, quadratic relationships, or relationships of the form $y = \frac{k}{x}$, based on rates of change in tables, symbolic forms, or graphical representations. Recognize that multiplying linear factors produces nonlinear relationships.
   a) Identify nonlinear (exponential, quadratic, and equations of the form $y = \frac{k}{x}$) relationships in graphical or tabular displays through an examination of successive differences, ratios, symbolic forms, or graphical properties.
   b) Identify terms in a geometric (exponential) sequence using verbal rules or symbolic expressions (explicit and recursive). Discuss the wording and notation.
   c) Compare (similarities and differences) multiplying numeric expressions versus multiplying algebraic expressions (e.g. binomial or trinomial expressions).
   d) Multiply a pair of linear expressions and interpret the result of the operation numerically by evaluation, through a table of values, and graphically.
   e) Recognize exponential functions from their verbal descriptions and tabular, graphical or symbolic representations, and translate among these representations.
   f) Describe the effects of changes in the coefficient, base, and exponent on the growth described by an exponential function.

2) Represent and interpret simple exponential and quadratic functions based on contextual phenomena using tables, symbolic forms, or graphical representations and solve equations related to these functions.
   a) Find integer powers of rational numbers; evaluate the meaning of integer powers of variables in expressions, and apply the basic laws of exponents (e.g., $a^m \bullet a^n = a^{m+n}$)
   b) Apply laws of exponents in problem situations (e.g. scientific notation).
   c) Distinguish among general representations for exponential equations ($y = b^x$, $y = a \cdot b^x$) and quadratic equations ($y = x^2$, $y = -x^2$, $y = ax^2$, $y = x^2 + c$, $y = ax^2 + c$), and describe how the values of *a*, *b*, and *c* affect their graphical and tabular representations.
   d) Provide and describe multiple representations of solutions to simple exponential and quadratic equations using manipulative models, verbal descriptions, tables, graphs, symbolic expressions, and technology.
   e) Factor simple quadratic expressions (limited to the removal of monomial terms, perfect-square trinomials, difference of squares, and quadratics of the form $x^2 + bx + c$ that factor over the integers), and apply the zero-product property to determine the solutions of the related equation.

* *Locally assessed items.*

Algebra 1, Core Concept D:
Design statistical studies, recognize the importance of random sampling, identify sources of bias, and quantify the strength of linear relationships.

Students design and conduct a survey based on an appropriate sample, and they interpret and communicate results. Students evaluate survey results reported in the media. Students discern the differences between random and nonrandom sampling methods and identify possible sources of bias in sampling. Students distinguish between sampling error and measurement error, and they understand that results may vary from sample to population and from sample to sample.

Learning Goals and Performance Indicators:

1) Formulate questions that can be addressed through collection and analysis of survey data. Explain the importance of random selection, and design and execute surveys. Use survey results to communicate an answer appropriate to the question of interest. Distinguish between sampling error and measurement error, and evaluate survey results reported in the media.

    a) Formulate a question of interest, and define key components that can be addressed through a survey. Define the population, the variables to measure, and how to measure the variables; identify factors that may influence survey outcomes; design questionnaires.
    b) Describe techniques for drawing simple random samples of members from a population. Identify situations in which a stratified random sample would be preferred over a simple random sample.
    c) Distinguish between a sample and a census, and explain the advantages and disadvantages of each.
    d) Design and implement the selection of a simple random sample; collect, organize, and display survey data in appropriate tables or graphs; and summarize the data using measures of center and spread, including the mean absolute deviation, and interpret the results obtained in the context of the question.
    e) Describe how the method of sample selection and the methods of measurement of outcomes can affect survey results, and explain how biases may arise from both sampling errors and measurement errors.

2) Understand that results may vary from sample to population and from sample to sample. Analyze, summarize, and compare results from random and nonrandom samples and from a census, using summary numbers and a variety of graphical displays to communicate findings.

    a) Compare measures of center and spread computed using sample data drawn from a population (sample statistics) with the same measures of center and spread computed using data from a census of the population (population parameters). Observe that sample means tend to approach the population mean as sample size increases.
    b) Recognize that measures of center and spread computed using data from a random sample are likely to differ from sample to sample even when the samples are drawn from the same population and have the same number of observations.
    c) Distinguish between random and nonrandom sampling methods, compare results from simple random and nonrandom samples drawn from the same population, and discuss how and why the results might differ because of potential sources of bias in the various samples.

* *Locally assessed items.*

Geometry, Core Concept A:
Apply the concepts of inductive and deductive reasoning to form or verify geometric conjectures.

Students represent geometric figures and investigate a variety of relationships among them, form conjectures, and attempt to verify or reject the conjectures. They develop and apply various methods of proving or disproving conjectures within the axiomatic structure of Euclidean geometry.

Learning Goals and Performance Indicators:

1) Describe and apply inductive and deductive reasoning to form conjectures and attempt to verify or reject them through developing short sequences of geometric theorems within a local axiomatic system or by developing counterexamples.

    a) Describe the structure of and relationships within an axiomatic system (undefined terms, defined terms, axioms/postulates, methods of reasoning, and theorems).
    b) Form conjectures based on exploring geometric situations with or without technology.
    c) Prove, directly or indirectly, that a valid mathematical statement is true. Develop a counterexample to refute an invalid statement.
    d) Develop the triangle angle-sum and angle-measure theorems for polygons, and the triangle- and angle-inequality theorems.
    e) Identify flaws or gaps in the reasoning supporting an argument.
    f) Formulate and investigate the validity of the converse, inverse, or contrapositive of a conditional statement.

2) Apply mathematical methods of proof to develop justifications for basic theorems of Euclidean geometry, including the Pythagorean Theorem and its converse.

    a) Justify statements about angles formed by perpendicular lines and transversals of parallel lines.
    b) Develop the triangle angle-sum and angle-measure theorems for polygons, and the triangle- and angle-inequality theorems.
    c) Justify and apply properties of circles (e.g., perpendicularity of tangent and radius, angle inscribed in a circle).
    d) Organize and present direct and indirect proofs using two-column, paragraph, and flow-chart formats.

3) Use a variety of representations to describe geometric objects and to analyze relationships among them.

    a) Use coordinates and algebraic representations (e.g., distances, points that divide segments in specified ratios, slope) to describe and define figures.
    b) Use nets, drawings (including isometric), vertex-edge graphs, models, and technologically created images to represent geometric objects from different perspectives and analyze relationships among them.

* *Locally assessed items.*

Geometry, Core Concept B:
Apply and analyze transformations of figures in the coordinate plane using properties of similarity and congruency.

Students identify the properties and perform the mappings of rigid transformations and *origin-centered* (needs defining) dilations of figures in the plane. They discuss the connections between rigid transformations and origin-centered dilations and the relations of congruence and similarity, respectively. They develop and apply sufficient conditions for proving planar figures congruent or similar. They apply similarity to solve a variety of contextual problems.

Learning Goals and Performance Indicators:

1) Identify and apply transformations of figures in the coordinate plane and discusses the results of these transformations. Identify and justify these congruences by establishing sufficient conditions and by finding a congruence-preserving rigid transformation between the figures. Solve problems involving congruence in a variety of contexts.

    a) Represent translations, line reflections, rotations, and origin-centered dilations of objects in the coordinate plane by using sketches, coordinates, and function notation, and explain the effects of these transformations and identify corresponding parts of congruent and similar figures after transformation(s).
    b) Recognize and identify corresponding parts of congruent and similar figures after transformation.
    c) Identify and differentiate among sufficient conditions for congruence of triangles (SSS, SAS, ASA, AAS, and HL) and apply them to identify congruent triangles.
    d) Use coordinate geometry and rigid transformations (reflections, translations, and rotations) to establish congruence of figures.

2) Identify similar figures and justify similarity by establishing sufficient conditions and by finding an origin-centered dilation between the figures. Solve problems involving similarity in a variety of contexts.

    a) Identify conditions for establishing similarity of triangles (SAS, SSS, AA), and apply them, noting that congruence is a special case of similarity.
    b) Create a representation of a figure similar to a specified figure given their similarity ratio and use origin-centered dilations to describe and investigate similarities.

* *Locally assessed items.*

Geometry, Core Concept C:
Justify and apply measurement formulas for two- and three-dimensional shapes including the Pythagorean Theorem and its applications.

Students justify and apply the measurement formulas associated with one-, two-, and three-dimensional geometric objects. They prove and apply the Pythagorean Theorem, right-triangle trigonometric ratios, and proportionality relationships in structuring and solving indirect measurement problems. Students extend their understanding of the real number system through solving problems in geometric situations.

Learning Goals and Performance Indicators:

1) Justify two- and three-dimensional measurement formulas for perimeter/circumference, area, and volume and apply these formulas and other geometric properties relating angle and arc measures to solving problems involving measures of simple and composite one-, two-, and three-dimensional geometric objects.

    a) Derive and justify the area formulas for quadrilaterals and regular polygons.
    b) Link the surface area of prisms and cylinders to the sum of the areas of their bases and lateral surfaces using planar nets to illustrate and sum the relevant measures.
    c) Identify and find the measures of angles formed by segments in three-dimensional figures, extending right-triangle and isosceles/equilateral-triangle relationships to study the planar faces of three-dimensional objects.
    d) Apply and link formulas to solve problems involving area, perimeter, volume, and surface area of prisms, cylinders, pyramids, cones, spheres, and composite figures.
    e) Apply the ratio of similitude to determine perimeter, area, and volume measurements of similar figures.
    f) Determine arc lengths of circles and areas of sectors of circles using proportions.

2) Apply the Pythagorean Theorem and its converse, develop and apply the distance formula, properties of special right triangles, properties of proportions, and the basic trigonometric ratios.

    a) Develop and apply the distance formula to determine the distance between points in the coordinate plane.
    b) Develop and apply the properties of 30°-60°-90° and 45°-45°-90° triangles.
    c) Develop and apply proportional relationships involving the altitude drawn to the hypotenuse of a right triangle.
    d) Apply, singly and in combination, the Pythagorean Theorem and its converse, properties of proportionality, and similarity in solving contextual problems in two- and three-dimensional settings.
    e) *Apply the sine, cosine, and tangent trigonometric ratios to determine lengths and angle measures in right triangles.

** Locally assessed items.*

Geometry, Core Concept D:
Explore two-stage experiments, conditional probability and independence and simulations of random phenomena.

Students determine the sample space for multi-stage experiments. They distinguish between independent and dependent events and compute their probabilities. Students design and use simulations to solve contextual probability problems. They extend their understanding of the real number system through problem solving situations.

Learning Goals and Performance Indicators:

1) Determine the sample space for multi-stage experiments using a variety of representations and employ systematic counting approaches to determine the number of possible outcomes. Distinguish between independent and dependent compound events, and compute their probabilities using a variety of representations and applying the multiplication rule for probability.

    a) Determine the number of possible outcomes in the sample space for multi-stage experiments using a variety of representations (e.g. tree diagrams, lists) and systematic counting approaches such as the multiplication counting rule.
    b) Distinguish between independent and dependent compound events, and explain the idea of conditional probability.
    c) Use trees, tables, area models, and other representational methods to describe and apply the multiplication rule for probability to compute probabilities for independent and for dependent compound events.

2) Develop, use, and interpret simulations to estimate probabilities for events where theoretical values are difficult or impossible to compute. Recognize that simulation results are likely to differ from one run of the simulation to the next, and that results of the simulation tend to converge as the number of trials increases.

    a) Describe a simulation by identifying the components and assumptions in a problem, selects a device to generate chance outcomes, defines a trial, and specify the number of trials; conduct a simulation.
    b) Summarize data from a simulation using appropriate graphical and numerical summaries, develop an estimate for the probability of an event associated with a contextual probabilistic situation where theoretical values are difficult or impossible to compute, and discuss the effect of the number of trials on the estimated probability of the event.
    c) Recognize that simulation results are likely to differ from one run of the simulation to the next; observes that the results of the simulation tend to converge as the number of trials increases.

* *Locally assessed items.*

Algebra 2, Core Concept A:
Represent, compare and translate polynomial equations and functions.

Students extend understanding of functions from linear settings to include polynomial functions, operations on these functions, and the solution of polynomial equations using complex numbers. They use polynomials (focus on quadratics) to model situations with graphical and symbolic representations. They translate among polynomial representations to represent and discuss the qualitative behavior of the associated functions. Students extend their understanding of the real number system through solving problems in algebraic situations.

Learning Goals and Performance Indicators:

1) Operate with monomials, binomials, and polynomials. Apply these operations to analyze the graphical behavior of polynomial functions. Apply the composition of functions to model and solve problems.
   a) Add, subtract, multiply, and divide polynomial expressions to solve problems.
   b) Analyze and describe graphs of polynomial functions by examining their intercepts, zeros, domain and range, and local (turning points) and global (end) behavior.
   c) Use factoring, properties of exponents, and knowledge of the related contextual needs (e.g. expanding to observe graphical behavior, understanding purpose and context of factoring) to transform expressions and solve problems.
   d) Apply the composition of functions to model and solve problems, and explain the results.

2) Represent, compare, and translate among representations (including graphic, symbolic, and tabular) to interpret, solve problems, and justify solutions involving quadratic functions.
   a) Identify, interpret, and translate among different representations of quadratic functions.
   b) Determine reasonable domain and range values for quadratic functions within a context, and test the reasonableness of solutions to quadratic equations (zeros of quadratic functions).
   c) Identify any points of intersection of the graph of a quadratic equation of the form $y = ax^2$ and the graph of a line of the form $y = k$, and relate the points of intersection to the solutions of the quadratic equation $ax^2 = k$.
   d) Sketch a quadratic function's graph, and recognize the relationships between the coefficients of a quadratic function and characteristics of its graph (e.g., shape, position, intercepts, zeros, maximum, minimum, symmetry, vertex).
   e) Formulate equations and inequalities based on quadratic functions and solve using factoring, completing the square, and/or technology. Interpret and justify solutions in terms of the original problem context.
   f) Develop the quadratic formula, and apply it to the solution of quadratic equations and the interpretation of the nature of their roots.
   g) Construct and solve quadratic inequalities in one and two variables, and represent their solutions graphically.

3) Represent, apply, and communicate understanding of the properties of complex numbers.
   a) Define, plot, and compute with complex numbers.
   b) Describe how the associative, commutative, and distributive properties of operations on real numbers extend to operations on complex numbers.
   c) Solve quadratic equations with real coefficients over the set of complex numbers.

* *Locally assessed items.*

Algebra 2, Core Concept B:
Model, represent and solve non-linear functions (exponential, logarithmic, and other functions).

Students develop exponential, logarithmic, and other non-linear functions (rational, radical, absolute value, and piecewise-defined) to represent, investigate, and solve problems in context. They extend their understanding of the real number system through solving problems in algebraic situations.

Learning Goals and Performance Indicators:

1) Represent geometric or exponential growth with exponential functions and equations, and apply such functions and equations to solve problems in contextual situations.
   a) Extend the properties of rational exponents to real exponents, relating expressions with rational exponents to the corresponding radical expressions.
   b) Describe the effects of changes in the coefficient, base, and exponent on the growth described by an exponential function.
   c) Approximate solutions to an exponential equation, and relate the solutions to the points of intersection of the graph of the exponential equation and the graph of a horizontal line.
   d) Analyze a problem situation modeled by an exponential function (e.g. exponential growth and decay, compound interest), formulate an equation or inequality, and solve the problem.
   e) Graph and analyze the behavior of exponential functions.

2) Define logarithmic functions and use them to solve problems in contextual situations.
   a) Define a logarithm as a solution to an exponential equation, and recognize the inverse relationship between functions defined by logarithms and exponential expressions, showing this relationship graphically.
   b) Solve problems by applying properties of logarithms (for example, $\log_b xy = \log_b x + \log_b y$) to construct equivalent forms of a logarithmic expression.
   c) Apply the inverse relationship between exponential and logarithmic functions to solve problems in context.

3) Interpret and represent rational and radical functions and solve rational and radical equations.
   a) Model and solve problems using direct, inverse, joint, and combined variation.
   b) Model problem situations by constructing equations and inequalities based on rational functions, use a variety of methods to solve them, and interpret solutions in terms of the problem situation.
   c) Add, subtract, multiply, divide, and evaluate rational functions and simplify rational expressions with linear and quadratic denominators.
   d) Describe the graphs of rational functions, describe limitations on the domains and ranges, and examine asymptotic behavior.
   e) Use properties of radicals to solve equations and identify extraneous roots when they occur.

4) Interpret and model step and other piecewise-defined (i.e. linear and quadratic) functions, including functions involving absolute value.
   a) Analyze a problem situation to determine or interpret reasonable domain and range values for piecewise-defined functions representing the situation.
   b) Interpret, construct and apply step functions (e.g., greatest integer/floor) and other piecewise-defined functions, including absolute value functions, to model and solve problems.
   c) Translates among verbal, graphical, tabular, and symbolic representations of step functions and other piecewise-defined functions, including absolute value functions.

* *Locally assessed items.*

Algebra 2, Core Concept C:
Model, construct and solve systems of equations and inequalities.

Students construct, solve, and interpret solutions to systems of linear equations and inequalities in two variables. They represent cross-categorized data in matrices and perform operations on matrices to model and interpret problem situations. They model and solve systems of equations with technology. Students extend their understanding of the real number system through solving problems in algebraic situations.

Learning Goals and Performance Indicators:

1) Construct, solve, and interpret solutions of systems of linear equations and inequalities in two variables representing contextual situations.

   a) Construct a system of linear equalities or inequalities in two variables to represent a contextual setting.
   b) Analyze and explain the reasoning used to solve systems of linear equations and inequalities in two variables.
   c) Solve a system of equalities or inequalities in two variables using a variety of methods, and interpret the meaning of the solution.

2) Represent and interpret cross-categorized data in matrices, develop properties of matrix addition, and use matrix addition and its properties to solve problems.

   a) *Represent numerical or relational data categorized by two variables in a matrix and label the rows and columns. Interpret the meaning of a particular entry in a matrix in terms of the labels of its row and column.
   b) *Use matrix row and column sums to analyze data.
   c) *Develop the properties of matrix addition, and add and subtract matrices to solve problems.

3) Multiply matrices, verify the properties of matrix multiplication, and use the matrix form for a system of linear equations to structure and solve systems consisting of two or three linear equations in two or three unknowns, respectively, with technology.

   a) *Verify the properties of matrix multiplication and multiplies matrices to solve problems.
   b) *Construct a system of linear equations modeling a contextual situation, and represent the system as a matrix equation ($A\boldsymbol{x} = \boldsymbol{b}$), that is,

$$\begin{aligned} ax + by &= c \\ dx + ey &= f \end{aligned} \quad \Leftrightarrow \quad \begin{bmatrix} a & b \\ d & e \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} c \\ f \end{bmatrix}.$$

   c) *Solve a system consisting of two or more linear equations in two or more unknowns, respectively, by solving the related matrix equation $Ax = b$, using technology to find $x = A^{-1}b$.

** Locally assessed items.*

Algebra 2, Core Concept D:
Distinguish among experiments, surveys, and observational studies.

Students distinguish among surveys, experiments, and observational studies. They design studies, collect and analyze data using appropriate methods, draw conclusions, and communicate results. They evaluate studies reported in the media.

Learning Goals and Performance Indicators:

1) Identify problems that can be addressed through collection and analysis of experimental data, design and implement simple comparative experiments, and draw appropriate conclusions from the collected data.

    a) Describe how well designed experiments use random assignment to balance the variation of some factors in order to isolate the effects of a treatment.
    b) Design a simple comparative experiment to answer a question: determine treatments, identify methods of measuring variables, randomly assign units to treatments, and collect data, distinguishing between explanatory and response variables.
    c) Organize and display data from an experiment; summarize the data using measures of center and spread, including the mean and standard deviation; identify patterns and trends in tables and plots; and communicate methods used and the results of the experimental study to non-technical persons.

2) Distinguish among surveys, observational studies, and designed experiments and relate each type of investigation to the research questions it is best suited to address. Recognize that an observed association between a response and an explanatory variable does not necessarily imply that the two variables are causally linked. Recognize the importance of random selection in surveys and random assignment in experimental studies. Communicate the purposes, methods, and results of a statistical study, and evaluate studies reported in the media.

    a) Distinguish among questions best explored through a sample survey, an observational study, or a designed experiment. Recognize that an observed association between a response and an explanatory variable does not necessarily imply that the two variables are causally linked.
    b) Formulate specific questions and identify quantitative measures that may be used in providing answers to the question of interest.
    c) Explain why most research questions do not have unique answers and why several approaches to answering the same question may be appropriate; explain how biases can occur in studies, why different studies of the same research question, conducted differently, may yield different results and why this is to be expected.
    d) Communicate the purposes, methods, and results of a statistical study using non-technical language.

** Locally assessed items.*

Algebra 2, Core Concept E:
Use permutations, combinations, and probability distributions to solve problems.

Students use permutations, combinations and the multiplication rule for counting to solve counting and related probability problems. Students recognize a binomial probability setting, develop and graph the binomial probability distribution, and compute the probability distribution for a binomial count. They extend their understanding of the real number system through solving problem situations.

Learning Goals and Performance Expectations:

1) Solve ordering, counting, and related probability problems. Recognize a binomial probability setting and compute the probability distribution for a binomial count.

    a) *Use permutations, combinations, and the multiplication rule for counting (Fundamental Property of Counting) to solve counting and probability problems.
    b) Recognize a binomial probability setting, and develop and graph the probability distribution for a binomial count.

2) Identify settings in which the normal distribution may be useful, and describe characteristics of the normal distribution. Use graphical displays and the empirical rule to solve problems.

    a) Identify settings in which the normal distribution may be useful, and describe characteristics of the normal distribution.
    b) Use graphical displays and the empirical rule to evaluate the appropriateness of the normal model for a given set of data, and use the empirical rule to estimate the probability that an event will occur in a specific interval that can be described in terms of whole numbers of standard deviations about the mean.

** Locally assessed items.*

Integrated I, Core Concept A:
Interpret, model, justify and compare linear and nonlinear relationships.

Students identify situations with a constant rate of change that can be represented by linear functions, model these situations using algebraic, graphical, tabular, and verbal representations and translate among them, interpret the models, and solve contextual problems. They compare (similarities and differences) linear and nonlinear relationships using rates of change and graphical representations. They apply the terminology and symbols associated with expressions, functions, and linear equations, including inputs, outputs, domain range, slope, intercepts, and independent and dependent variables.

Learning Goals and Performance Indicators:

1) Identify functions based on their graphical behavior and rates of change, and describes functions using appropriate notation and terminology.

    a) Determine and justify whether a relationship is a function by using a graph or a verbal description of the relationship.
    b) Compare and justify linear or nonlinear relationships based on rate of change, verbal description, table of values, graphical representation, or symbolic form.
    c) Describe characteristics of piecewise-linear functions, including absolute value, and situations in which they arise.

2) Use linear functions to interpret, model, and solve situations having a constant rate of change. Use numeric patterns to generate symbolic patterns.

    a) Generalize linear patterns or arithmetic sequences using verbal rules and symbolic expressions such as kx and $ax + b$ in representing proportional or more-general linear relationships, respectively.
    b) Analyze a contextual situation; determine whether the situation can be described by a linear model, and if so, determine the constant rate of change and develop and interpret a linear function to model that situation.

* *Locally assessed items.*

Integrated 1, Core Concept B:
Interpret, represent and solve linear equations and inequalities.

Students distinguish among different uses of variables and find equivalent expressions and equations. They construct, represent, solve, and interpret solutions of linear equations, linear inequalities, and systems of linear equations for contextual situations. They determine the reasonableness of the solution(s) and extend their understanding of the real number system through solving problems in algebraic situations.

Learning Goals and Performance Indicators:

1) Represent linear patterns using expressions, equations, functions, and inequalities and interpret the meanings of these representations, recognizing which are equivalent and which are not.

    a) Describe the meaning of symbolic expressions and functions of the form $ax + b$ and f(x)=ax +b in words, and interpret the changes resulting from different values of the parameters a and b.
    b) Develop and justify equivalent algebraic expressions, equations, and inequalities using the properties of equality and inequality, as well as the commutative, associative, inverse, identity, and distributive properties.
    c) Identify and translate among equivalent representations of linear expressions, equations, inequalities, and systems of equations, using verbal, tabular, graphical, and symbolic representations.
    d) Write, interpret, and translate among equivalent forms of linear equations and functions, including slope-intercept, point-slope, intercept, and general (standard) forms, recognizing that equivalent forms for a linear relationship reveal more or less information about a given situation.
    e) Judge whether a scatterplot appears to show a linear trend, and if it does, draw a trend line and write an equation for that line; use the equation to make predictions; and interpret the slope of the line in context.

2) Distinguish among the different uses of variables, parameters, constants, and equations to construct, solve, and interpret solutions of linear equations and inequalities representing contextual situations.

    a) Identify and distinguish among parameters and the independent and dependent variables.
    b) Describe and distinguish among the types of equations that can be constructed by equating linear expressions, including identities (e.g., $x + x = 2x$); equations for which there is no solution (e.g., $x + 1 = x + 2$); formulas (e.g., $C = \pi d$); equations where the solution is unique (e.g., $2x + 3 = 16$); and equations relating two variables (e.g., $y = 3x + 7$).
    c) Identify terms in an arithmetic (linear) sequence using verbal rules or symbolic expressions (explicit and recursive).
    d) Construct a linear equation or linear inequality to model a contextual situation, using a variety of methods and representations.
    e) Analyze and explain the reasoning used to solve linear equations and linear inequalities using symbolic methods, graphs, tables, and technology.

* *Locally assessed items.*

Integrated 1, Core Concept C:
Model, interpret, represent and solve nonlinear functions.

Students identify and classify non-linear relationships. They represent simple contextual phenomena using exponential and quadratic functions and solve equations involving these functions with a variety of techniques. They determine the reasonableness of the solution(s). Students extend their understanding of the real number system through solving problems in algebraic situations.

Learning Goals and Performance Indicators:

1) Identify certain nonlinear relationships and classify them as exponential relationships, quadratic relationships, or relationships of the form $y = \frac{k}{x}$, based on rates of change in tables, symbolic forms, or graphical representations. Recognize that multiplying linear factors produces nonlinear relationships.
   a) Identify nonlinear (exponential, quadratic, and equations of the form $y = \frac{k}{x}$) relationships in graphical or tabular displays through an examination of successive differences, ratios, symbolic forms, or graphical properties.
   b) Identify terms in a geometric (exponential) sequence using verbal rules or symbolic expressions (explicit and recursive). Discuss the wording and notation.
   c) Compare (similarities and differences) multiplying numeric expressions versus multiplying algebraic expressions (e.g. binomial or trinomial expressions).
   d) Multiply a pair of linear expressions, and interpret the result of the operation numerically by evaluation, through a table of values, and graphically.

2) Represent and interpret simple exponential and quadratic functions based on contextual phenomena using tables, symbolic forms, or graphical representations and solves equations related to these functions.
   a) Find integer powers of rational numbers; evaluate the meaning of integer powers of variables in expressions, and apply basic laws of exponents (for example, $a^m \bullet a^n = a^{m+n}$).
   b) Recognize exponential functions from their verbal descriptions and tabular, graphical or symbolic representations, and translate among these representations.
   c) Describe the effects of changes in the coefficient, base, and exponent on the growth described by an exponential function.
   d) Apply laws of exponents in problem situations (e.g. scientific notation).
   e) Distinguish among general representations for exponential equations ($y = b^x$, $y = a \cdot b^x$) and quadratic equations ($y = x^2$, $y = -x^2$, $y = ax^2$, $y = x^2 + c$, $y = ax^2 + c$), and describe how the values of *a*, *b*, and *c* affect their graphical and tabular representations.
   f) Provide and describe multiple representations of solutions to simple exponential and quadratic equations using manipulative models, verbal descriptions, tables, graphs, symbolic expressions, and technology.
   g) Factor simple quadratic expressions (limited to the removal of monomial terms, perfect-square trinomials, difference of squares, and quadratics of the form $x^2 + bx + c$ that factor over the integers), and apply the zero-product property to determine the solutions of the related equation.

** Locally assessed items.*

Integrated 1, Core Concept D:
Represent, develop and apply patterns of shape, geometric reasoning, and geometric relationships.

Students represent geometric objects and investigate a variety of relationships among them, form conjectures and attempt to verify or reject the conjectures. They develop and apply various methods of proving statements or disproving conjectures within the axiomatic structure of Euclidean geometry.

Learning Goals and Performance Indicators:

1) Use a variety of representations to describe geometric objects and to analyze relationships among them.

    a) *Use coordinates and algebraic representations (e.g., distances, points that divide segments in specified ratios, slope) to describe and define figures.
    b) *Use nets, drawings (including isometric), vertex-edge graphs, models, and technologically created images to represent geometric objects from different perspectives and analyze relationships among them.

2) Develop, test, and provide justifications, based on inductive and deductive methods, for conjectures involving relations of lines, angles, and figures.

    a) *Describe the structure of and relationships within an axiomatic system (undefined terms, defined terms, axioms/postulates, methods of reasoning, and theorems).
    b) *Recognize flaws or gaps in the reasoning supporting an argument.
    c) *Develop and test conjectures about angles, lines, bisectors, polygons (especially triangles and quadrilaterals), circles, and three-dimensional figures.
    d) *Justify statements about angles formed by perpendicular lines and transversals of parallel lines.

** Locally assessed items.*

Integrated 1, Core Concept E:
Design statistical studies, recognize the importance of random sampling, identify sources of bias, and quantifying the strength of linear relationships.

Students design and conduct a survey based on an appropriate sample, and they interpret and communicate results. Students evaluate survey results reported in the media. They discern the differences between random and nonrandom sampling methods and identify possible sources of bias in sampling. Students distinguish between sampling error and measurement error, and they understand that results may vary from sample to population and from sample to sample.

Learning Goals and Performance Indicators:

1) Formulate questions that can be addressed through collection and analysis of survey data. Explain the importance of random selection, and design and execute surveys. Use survey results to communicate an answer appropriate to the question of interest. Distinguish between sampling error and measurement error, and evaluate survey results reported in the media.

    a) Formulate a question of interest, and define key components that can be addressed through a survey. Define the population, the variables to measure, and how to measure the variables; identify factors that may influence survey outcomes; design questionnaires.
    b) Describe techniques for drawing simple random samples of members from a population. Identify situations in which a stratified random sample would be preferred over a simple random sample.
    c) Distinguish between a sample and a census, and explain the advantages and disadvantages of each.
    d) Design and implement the selection of a simple random sample; collect, organize, and display survey data in appropriate tables or graphs; and summarize the data using measures of center and spread, including the mean absolute deviation, and interpret the results obtained in the context of the question.
    e) Describe how the method of sample selection and the methods of measurement of outcomes can affect survey results, and explain how biases may arise from both sampling errors and measurement errors.

2) Understand that results may vary from sample to population and from sample to sample. Analyze, summarize, and compare results from random and nonrandom samples and from a census, using summary numbers and a variety of graphical displays to communicate findings.

    a) Compare measures of center and spread computed using sample data drawn from a population (sample statistics) with the same measures of center and spread computed using data from a census of the population (population parameters). Observe that sample means tend to approach the population mean as sample size increases.
    b) Recognize that measures of center and spread computed using data from a random sample are likely to differ from sample to sample even when the samples are drawn from the same population and have the same number of observations.
    c) Distinguish between random and nonrandom sampling methods, compare results from simple random and nonrandom samples drawn from the same population, and discuss how and why the results might differ because of potential sources of bias in the various samples.

* *Locally assessed items.*

INTEGRATED 2

Integrated 2, Core Concept A:
Model and solve systems of linear equations and matrices.

Students represent relationships that can be modeled by a system of linear equations and solve the system using a variety of methods and representations. They represent and interpret data and systems of equations through matrix representations, using addition and multiplication of matrices as appropriate. They use matrix equations and inverses, where they exist, to find solutions to systems of equations using technology. Students extend their understanding of the real number system through solving problems in algebraic situations.

Learning Goals and Performance Indicators:

1) Represent relationships that can be modeled by a system of linear equations and inequalities and solve the system using a variety of methods and representations.

   a) Construct a system of linear equations or inequalities modeling a contextual situation, using a variety of methods and representations.
   b) Analyze and explain the reasoning used to solve a system of linear equations or inequalities.
   c) Solve a system consisting of two linear equations or inequalities in two unknowns, using graphs, tables, symbolic methods, and technology, and describe the nature of the solutions (no solution, one solution, infinitely many solutions).

2) Represent and interpret cross-categorized data in matrices, develop properties of matrix addition, and use matrix addition and its properties to solve problems.

   a) Represent numerical or relational data categorized by two variables in a matrix and labels the rows and columns. Interpret the meaning of a particular entry in a matrix in terms of the labels of its row and column.
   b) Use matrix row and column sums to analyze data.
   c) Develop the properties of matrix addition, and add and subtract matrices to solve problems.

3) Multiply matrices, verify the properties of matrix multiplication, and use the matrix form for a system of linear equations to structure and solve systems consisting of two or three linear equations in two or three unknowns, respectively, with technology.

   a) Verify the properties of matrix multiplication and multiply matrices to solve problems.
   b) Construct a system of linear equations modeling a contextual situation, and represent the system as a matrix equation ($A\mathbf{x} = \mathbf{b}$), that is, $\begin{aligned} ax + by &= c \\ dx + ey &= f \end{aligned} \Leftrightarrow$

   $$\begin{bmatrix} a & b \\ d & e \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} c \\ f \end{bmatrix}.$$

   c) Solve a system consisting of two or three linear equations in two or three unknowns, respectively, by solving the related matrix equation $Ax = b$, using technology to find $x = A^{-1}b$

* *Locally assessed items.*

Integrated 2, Core Concept B:
Develop and apply geometric proof, similarity, and transformations.

Students apply and analyze transformations of figures in the coordinate plane using properties of similarity and congruency and apply the concepts of inductive and deductive reasoning to form or verify conjectures. They develop general methods of proof and apply these methods to solve problems involving congruence, similarity, rigid transformations, and origin-centered dilations of figures in the plane.

Learning Goals and Performance Indicators:

1) Apply mathematical methods of proof to develop justifications for basic theorems of Euclidean geometry.
   a) Form conjectures based on exploring geometric situations with or without technology.
   b) Prove, directly or indirectly, that a valid mathematical statement is true. Develop a counterexample to refute an invalid statement.
   c) Formulate and investigate the validity of the converse of a conditional statement.
   d) Organize and present direct and indirect proofs using two-column, paragraph, and flow-chart formats.

2) Identify congruent figures and justify these congruences by establishing sufficient conditions and by finding a congruence-preserving transformation between the figures. Solve problems involving congruence in a variety of contexts.
   a) Analyze figures in terms of their symmetries using the concepts of reflection, rotation, and translation and combinations of these.
   b) Compare and contrast equality, congruence, and similarity.
   c) Identify and differentiate among sufficient conditions for congruence of triangles (SSS, SAS, ASA, AAS, and HL).
   d) Use coordinate geometry and rigid transformations (reflections, translations, and rotations) to establish congruence of figures.

3) Identify and apply transformations of figures in the coordinate plane and discuss the results of these transformations.
   a) Represent translations, line reflections, rotations, and origin-centered dilations of objects in the coordinate plane by using sketches, coordinates, and function notation, and explain the effects of these transformations.
   b) Recognize and identify corresponding parts of congruent and similar figures after transformation.

4) Identify similar figures and justify these similarities by establishing sufficient conditions and by finding a similarity-preserving rigid transformation or origin-centered dilation between the figures. Solve problems involving similarity in a variety of contexts.
   a) Identify conditions (SAS, SSS, and AA) for establishing similarity of triangles, and apply them, noting that congruence is a special case of similarity.
   b) Use similarity to calculate the measures of corresponding parts of similar figures, and apply similarity in a variety of problem-solving contexts within mathematics and other disciplines.
   c) Create a representation of a figure similar to a specified figure given their similarity ratio.
   d) Use similar triangles to demonstrate that the rate of change associated with any pair of points on a line is the same.
   e) Use origin-centered dilations to describe and investigate similarities.

* *Locally assessed items.*

Integrated 2, Core Concept C:
Justify and apply measurement formulas for two- and three-dimensional shapes.

Students justify and apply the measurement formulas associated with one-, two-, and three-dimensional geometric objects. Students extend their understanding of the real number system through solving problems in geometric situations.

Learning Goals and Performance Indicators:

1) Justify two- and three-dimensional measurement formulas for perimeter/circumference, area, and volume and apply these formulas and other geometric properties relating angle and arc measures to solving problems involving measures of simple and composite one-, two-, and three-dimensional geometric objects.

    a) Justify the area formulas for quadrilaterals and regular polygons.
    b) Apply the (volume) = (area of the base) × (height) principle in linking area and volume formulas for prisms and cylinders.
    c) Link the surface area of prisms and cylinders to the sum of the areas of their bases and lateral surfaces using planar nets to illustrate and sum the relevant measures.
    d) Identify and find the measures of angles formed by segments in three-dimensional figures, extending right-triangle and isosceles/equilateral triangle relationships to study the planar faces of three-dimensional objects.
    e) Apply formulas and solve problems involving area, perimeter, volume, and surface area of pyramids, cones, spheres, and composite figures.
    f) Determine arc lengths of circles and areas of sectors of circles using proportions.
    g) Develop the triangle angle-sum and angle-measure theorems for polygons, and the triangle- and angle-inequality theorems.
    h) Justify and apply statements about angles formed by chords, tangents, and secants in circles and the measures of their intercepted arcs.

* *Locally assessed items.*

Integrated 2, Core Concept D:
Explore two-stage experiments, conditional probability and independence, and simulations of random phenomena.

Students determine the sample space for multi-stage experiments. They distinguish between independent and dependent events and compute their probabilities. Students design and use simulations to solve contextual probability problems. Students extend their understanding of the real number system through solving problem situations.

Learning Goals and Performance Indicators:

1) Determine the sample space for multi-stage experiments using a variety of representations and employ systematic counting approaches to determine the number of possible outcomes. Distinguish between independent and dependent compound events, and compute their probabilities using a variety of representations and applying the multiplication rule for probability.

    f) Determine the number of possible outcomes in the sample space for multi-stage experiments using a variety of representations (e.g. tree diagrams, lists) and systematic counting approaches such as the multiplication counting rule.
    g) Distinguish between independent and dependent compound events, and explain the idea of conditional probability.
    h) Use trees, tables, area models, and other representational methods to describe and apply the multiplication rule for probability to compute probabilities for independent and for dependent compound events.

2) Develop, use, and interpret simulations to estimate probabilities for events where theoretical values are difficult or impossible to compute. Recognize that simulation results are likely to differ from one run of the simulation to the next, and that results of the simulation tend to converge as the number of trials increases.

    a) Describe a simulation by identifying the components and assumptions in a problem, select a device to generate chance outcomes, defines a trial, and specifies the number of trials; conduct a simulation.
    b) Summarize data from a simulation using appropriate graphical and numerical summaries, develop an estimate for the probability of an event associated with a contextual probabilistic situation where theoretical values are difficult or impossible to compute, and discuss the effect of the number of trials on the estimated probability of the event.
    c) Recognize that simulation results are likely to differ from one run of the simulation to the next; observe that the results of the simulation tend to converge as the number of trials increases.

* *Locally assessed items.*

INTEGRATED 3

Integrated 3, Core Concept A:
Represent, compare, and translate polynomial equations and functions.

Students extend understanding of functions from linear settings to include polynomial functions, operations on these functions, and the solution of polynomial equations using complex numbers. They use polynomials, with emphasis on quadratics, to model situations with graphical and symbolic representations. They translate among polynomial representations to represent and discuss the qualitative behavior of the associated functions.

Learning Goals and Performance Indicators:

1) Operate with monomials, binomials, and polynomials, apply these operations to analyze the graphical behavior of polynomial functions, and apply the composition of functions to model and solve problems.

    a) Add, subtract, multiply, and divide polynomial expressions to solve problems.
    b) Analyze and describe graphs of polynomial functions by examining their intercepts, zeros, domain and range, and local (turning points) and global (end) behavior.
    c) Use factoring, properties of exponents, and knowledge of the related contextual needs to transform expressions and solve problems.
    d) Apply the composition of functions to model and solve problems, explaining the results.

2) Represent, compare, translate among representations (including graphic, symbolic, and tabular) to interpret, solve problems, and justify solutions involving quadratic functions.

    a) Identify, interpret, and translate among different representations of quadratic functions.
    b) Determine reasonable domain and range values for quadratic functions within a context, and test the reasonableness of solutions to quadratic equations (zeros of quadratic functions).
    c) Identify any points of intersection of the graph of a quadratic equation of the form $y = ax^2$ and the graph of a line of the form $y = k$, and relate the points of intersection to the solutions of the quadratic equation $ax^2 = k$.
    d) Sketch a quadratic function's graph, and recognize the relationships between the coefficients of a quadratic function and characteristics of its graph (e.g., shape, position, intercepts, zeros, maximum, minimum, symmetry, vertex).
    e) Formulate equations and inequalities based on quadratic functions, solve (using factoring, completing the square, and technology) to interpret and justify their solutions in terms of the original problem context.
    f) Develop the quadratic formula, and apply it to the solution of quadratic equations and the interpretation of the nature of their roots.
    g) Construct and solve quadratic inequalities in one and two variables, and represent their solutions graphically.

3) Represents, apply, and discuss the properties of complex numbers.

    a) Define, plot, and compute with complex numbers.
    b) Describe how the associative, commutative, and distributive properties of operations on real numbers extend to operations on complex numbers.
    c) Solve quadratic equations with real coefficients over the set of complex numbers.

** Locally assessed items.*

Integrated 3, Core Concept B:
Model, represent, and solve non-linear functions (exponential, logarithmic, and other functions).

Students develop exponential, logarithmic, and other non-linear functions (rational, radical, absolute value, and piecewise-defined) to represent, investigate, and solve problems in context. They extend their understanding of the real number system through solving problems in algebraic situations.

Learning Goals and Performance Indicators:

1) Represent geometric or exponential growth with exponential functions and equations, and apply such functions and equations to solve problems in context.
   a) Extend the properties of rational exponents to real exponents, relating expressions with rational exponents to the corresponding radical expressions.
   b) Approximate solutions to an exponential equation, and relate the solutions to the points of intersection of the graph of the exponential equation and the graph of a horizontal line.
   c) Analyze a problem situation modeled by an exponential function (e.g. exponential growth and decay, compound interest), formulate an equation or inequality, and solve the problem.
   d) Graph and analyze the behavior of exponential functions.

2) Defines logarithmic functions and uses them to solve problems in contextual situations.
   a) Define logarithm as a solution to an exponential equation, and recognize the inverse relationship between functions defined by logarithms and exponential expressions, showing this relationship graphically.
   b) Solve problems by applying properties of logarithms (for example, $\log_b xy = \log_b x + \log_b y$) to construct equivalent forms of a logarithmic expression.
   c) Apply the inverse relationship between exponential and logarithmic functions to solve problems in contextual situations.

3) Interpret and represent rational and radical functions and solve rational and radical equations.
   a) Model and solve problems using direct, inverse, joint, and combined variation.
   b) Model problem situations by constructing equations and inequalities based on rational functions, use a variety of methods to solve them, and interpret the solutions in terms of the problem situation.
   c) Add, subtract, multiply, and evaluate rational functions and simplify rational expressions with linear and quadratic denominators.
   d) Describe the graphs of rational functions, describe limitations on the domains and ranges, and examine asymptotic behavior.
   e) Use properties of radicals to solve equations and identify extraneous roots when they occur.

4) Interpret and model step and other piecewise-defined (i.e. linear and quadratic) functions, including functions involving absolute value.
   a) Analyze a problem situation to determine or interpret reasonable domain and range values for piecewise-defined functions representing the situation.
   b) Interpret, construct, and apply step functions (e.g., greatest integer/floor) and other piecewise-defined functions, including absolute value functions, to model and solve problems.
   c) Translate among verbal, graphical, tabular, and symbolic representations of step functions and other piecewise-defined functions, including absolute value functions.

** Locally assessed items.*

Integrated 3, Core Concept C:
Develop, analyze and model sequences and recursion.

Student analyzes and represents sequences and series and investigates how recursive relationships and their associated sequences can model the long-term behavior of situations involving sequential change. Students extend their understanding of the real number system through solving problems in algebraic situations.

Learning Goals and Performance Indicators:

1) Categorize sequences as arithmetic, geometric, or neither and develop formulas for the general terms and sums related to arithmetic and geometric sequences.

    a) Investigate the rate of change found in sequences, and use it to characterize sequences as arithmetic, geometric, or neither.
    b) Develop the general term for arithmetic and geometric sequences and develop methods for calculating sums of terms for finite arithmetic and geometric sequences and the sum of a convergent infinite geometric series.

2) Develop recursive relationships for modeling and investigating patterns in the long-term behavior of their associated sequences.

    a) Develop recursive relationships for arithmetic and for geometric growth situations.
    b) Generate or construct sequences from given recursive relationships modeling patterns found in mathematics and in other disciplines.
    c) Investigate the long-term behavior of a recursive relationship, with and without technology.

** Locally assessed items.*

Integrated 3, Core Concept D:
Apply the Pythagorean Theorem and trigonometric ratios to indirect measurement situations.

Student develops and applies the Pythagorean Theorem, right-triangle trigonometric ratios, and proportionality relationships in structuring and solving indirect measurement problems

Learning Goals and Performance Indicators:

1) Prove and apply the Pythagorean Theorem and its converse, and develop and apply the distance formula, properties of special right triangles, properties of proportions, and the basic trigonometric ratios.

    a) *Prove the Pythagorean Theorem and its converse.
    b) Develop and apply the distance formula to determine the distance between points in the coordinate plane.
    c) Develop and apply the properties of 30°-60°-90° and 45°-45°-90° triangles; develop and apply proportional relationships involving the altitude drawn to the hypotenuse of a right triangle.
    d) Apply the sine, cosine, and tangent trigonometric ratios to determine lengths and angle measures in right triangles.
    e) Apply, singly and in combination, the Pythagorean Theorem and its converse, properties of proportionality, and similarity in solving contextual problems in two- and three-dimensional settings.

** Locally assessed items.*

Integrated 3, Core Concept E:
Distinguish among experiments, surveys, and observational studies.

Students distinguish among surveys, experiments, and observational studies. They design studies, collect and analyze data using appropriate methods, draw conclusions, and communicate results. They evaluate studies reported in the media.

Learning Goals and Performance Indicators:

1) Identify problems that can be addressed through collection and analysis of experimental data, design and implement simple comparative experiments, and draw appropriate conclusions from the collected data.

    a) Describe how well designed experiments use random assignment to balance the variation of some factors in order to isolate the effects of a treatment.
    b) Design a simple comparative experiment to answer a question: determine treatments, identify methods of measuring variables, randomly assign units to treatments, and collect data, distinguishing between explanatory and response variables.
    c) Organize and display data from an experiment; summarize the data using measures of center and spread, including the mean and standard deviation; identify patterns and trends in tables and plots; and communicate methods used and the results of the experimental study to non-technical persons.

2) Distinguish among surveys, observational studies, and designed experiments and relate each type of investigation to the research questions it is best suited to address. Recognize that an observed association between a response and an explanatory variable does not necessarily imply that the two variables are causally linked. Recognize the importance of random selection in surveys and random assignment in experimental studies. Communicate the purposes, methods, and results of a statistical study, and evaluate studies reported in the media.

    a) Distinguish among questions best explored through a sample survey, an observational study, or a designed experiment. Recognize that an observed association between a response and an explanatory variable does not necessarily imply that the two variables are causally linked.
    b) Formulate specific questions and identify quantitative measures that may be used in providing answers to the question of interest.
    c) Explain why most research questions do not have unique answers and why several approaches to answering the same question may be appropriate; explain how biases can occur in studies, why different studies of the same research question, conducted differently, may yield different results and why this is to be expected.
    d) Communicate the purposes, methods, and results of a statistical study using non-technical language.

** Locally assessed items.*

Integrated 3, Core Concept F:
Use permutations, combinations, and probability distributions to solve problems.

Students use permutations, combinations and the multiplication rule for counting to solve counting and related probability problems. Students recognize a binomial probability setting, develop and graph the binomial probability distribution, and compute the probability distribution for a binomial count. They extend their understanding of the real number system through solving problem situations.

Learning Goals and Performance Indicators:

1) Solve ordering, counting, and related probability problems. Recognize a binomial probability setting and compute the probability distribution for a binomial count.

    a) *Use permutations, combinations, and the multiplication rule for counting (Fundamental Property of Counting) to solve counting and probability problems.
    b) Recognize a binomial probability setting, and develop and graph the probability distribution for a binomial count.

2) Identify settings in which the normal distribution may be useful, and describe characteristics of the normal distribution. Use graphical displays and the empirical rule to solve problems.

    a) Identify settings in which the normal distribution may be useful, and describe characteristics of the normal distribution.
    b) Use graphical displays and the empirical rule to evaluate the appropriateness of the normal model for a given set of data, and use the empirical rule to estimate the probability that an event will occur in a specific interval that can be described in terms of whole numbers of standard deviations about the mean.

** Locally assessed items.*

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