

**Missouri K-12 Mathematics:
Core Content, Learning Goals and
Performance Indicators**

Revised DRAFT

Spring, 2009

Missouri Department of Elementary and Secondary Education

Mathematics, Engineering, Technology and Science (METS) Alliance

TABLE OF CONTENTS

| | |
|---|----|
| Preface | i |
| Introduction | 1 |
| References and Primary Resources | 5 |
| Summary of Core Content, Kindergarten through High School | 7 |
| Core Content, Learning Goals and Performance Indicators (Grades K – 5) | 10 |
| Grade K | 11 |
| Grade 1 | 16 |
| Grade 2 | 20 |
| Grade 3 | 25 |
| Grade 4 | 30 |
| Grade 5 | 35 |
| Core Content, Learning Goals and Performance Indicators (Grades 6-8) | 40 |
| Grade 6 | 41 |
| Grade 7 | 47 |
| Grade 8 | 52 |
| Core Content, Learning Goals and Performance Indicators (High School) | 58 |
| Algebra I | 59 |
| Geometry | 64 |
| Algebra II | 69 |
| Pre-Calculus (to be added after further development, public review and revision) | |
| Integrated Mathematics I | 74 |
| Integrated Mathematics II | 80 |
| Integrated Mathematics III | 85 |
| Integrated Mathematics IV (to be added after further development, public review and revision) | |
| Glossary (see DRAFT at: http://dese.mo.gov/divimprove/curriculum/math/ Additional content to be added) | |

PREFACE

Since the Outstanding Schools Act of 1993, several documents have been developed to aid Missouri school districts in organizing curriculum to enable all students to achieve their potential. Most prominently, the *Show-Me Standards* (1996) identify broad content knowledge and process skills for all students to be successful as they continue their education, enter the workforce, and assume civic responsibilities.

In response to the federal *No Child Left Behind Act* of 2001, the Missouri Department of Elementary and Secondary Education created the first set of K-12 mathematics grade-level expectations (GLEs), published in 2004. This document, updated to version 2.0 in 2007, conveyed to Missouri educators and the general public mathematics assessment specifications for specific grades and courses, K-12. That is, it conveys the mathematics that students should know and will be assessed annually via the Missouri Assessment Program (MAP). However, the GLEs are not a comprehensive curriculum framework. They are not intended to specify all the mathematics content that should be taught at each grade level or within each course or the extent to which specific mathematics content should be emphasized.

Rationale for the Development of K-12 Mathematics Learning Goals

In 2007, three factors led to the development of the *K-12 Mathematics Learning Goals*, a curriculum framework that specifies both what students should learn and what will be assessed on annual MAP assessment. First, Governor Matt Blunt hosted a summit of key business stakeholders focused on ensuring that Missouri would continue to compete in the global market calling for increased skills in the areas of mathematics, engineering, technology, and science. A recommendation of Summit participants called for a revision of Missouri's K-12 GLEs and assessments for mathematics and science to support instruction of important mathematics content and process skills.

Second, the Missouri State Board of Education authorized a change in the required high school annual assessment – moving from a general grade 10 mathematics assessment to a series of end-of-course assessments in mathematics, beginning with Algebra I in 2008-09. This change necessitated a revision of the 2004 Missouri GLE document from a grade-based framework to a course-based framework at the high school level. At about the same time, state legislation led to work by the Department of Higher Education (DHE) to articulate college entrance expectations in mathematics and other subjects.

Third, national attention focused on strategies to strengthen K-12 mathematics programs. The commonly held belief that the U.S. mathematics curriculum lacks focus, coherence and depth documented by the Trends in International Mathematics and Science Study (TIMSS) provided the impetus for additional work in defining curriculum standards/learning goals. In response, several national organizations (Achieve, Inc., the College Board, the National Council of Teachers of Mathematics, and the American Statistical Association) developed model K-12 curriculum standards for mathematics and/or statistics. In March of 2008, the National Mathematics Advisory Panel made recommendations regarding the mathematics content needed to learn algebra and be prepared for studying higher levels of mathematics. Thus, a wealth of new information and recommendations regarding the K-12 school mathematics curriculum were available in 2007-08 to guide local and state curriculum work.

These factors led to collaboration between the Missouri Department of Elementary and Secondary Education (DESE) and the Mathematics, Engineering, Technology and Science (METS) Alliance. As a joint initiative, DESE and METS collaborated to develop kindergarten through high school mathematics curriculum goals that specify priorities (core content), learning goals and performance indicators (assessable content) for each grade (K-8) and core high school courses (Algebra I, Geometry, Algebra II, Integrated Mathematics I, II, and III). The *Missouri K-12 Mathematics Learning Goals* is the result of that collaboration and an open process of review from the general public, teachers, school administrators and university faculty.

Overview of the Project

Beginning in November 2007 a writing group comprised of Missouri mathematics teachers, curriculum specialists, and university faculty reviewed curriculum recommendations by national groups including Achieve, Inc., the College Board, the American Statistical Association, and the National Council of Teachers of Mathematics. Each group made recommendations about mathematics curriculum emphasis and organization based on extensive analysis of the needs of today's workforce and preparation for college. At the same time, the National Mathematics Advisory Panel (NMAP) commissioned by President George W. Bush was developing recommendations for K-12 school mathematics. Together with the Missouri GLEs (v. 2.0) and the National Assessment of Educational Progress (NAEP) framework, the recommendations of all of the national groups were used to craft the curriculum recommendations in this document.

The primary purpose of the *Missouri K-12 Mathematics Learning Goals* to articulate a focused, coherent, and forward-looking mathematics program to prepare ALL Missouri students for careers and livelihood in today's technology- and statistics-rich environment. Care was taken to outline a few content priorities (core content) per grade level or course rather than to cover a wide variety of topics across all mathematics strands. This approach allows for depth of learning, yet addresses all of the major goals across K-12.

Although the core content and learning goals specified in this document are intended for ALL students, some students are likely to advance through the outlined content more quickly than others. In fact, it is common practice in many Missouri schools for some students in middle school to begin the formal study of what has traditionally been high school mathematics. Local educational agencies are encouraged to develop and implement policies for serving their students including those that are ready for early advancement. While early advancement is appropriate for some students, this document outlines the mathematics content that all Missouri students should have an opportunity to learn over their K-12 schooling.

In addition to specification of content emphasis, this document emphasizes the development of important mathematical processes such as problem solving, reasoning, communication, connections, and representations. Without these processes, the skills and understandings outlined in the content strands are of limited value.

Writing Group

The *Missouri K-12 Mathematics Learning Goals* was developed through an open process over a 15-month period which included resource review, discussion, draft writing, public and commissioned review, and revision. The Writing Group consisted of respected Missouri K-12 teachers, administrators, and university educators with assistance from staff at DESE and METS. Members of the writing group have undergraduate and masters degrees in mathematics, mathematics education, elementary education, and/or school administration. They are Presidential Award winners, National Board certified teachers, Milken Award recipients, and former Presidents of the Missouri Council of Teachers of Mathematics. They have nearly 400 years of combined teaching experience, K-12. The individuals who played key roles in this project include:

Writing Group Co-Chair: Cindy Bryant, Missouri DESE
Writing Group Co-Chair: Barbara Reys, University of Missouri
METS Representative: Vicki May, Washington University

| | |
|--|--|
| <u>Grades K-5 Writing Team</u> Cathy Fueglein, Chair, Jennings School District Kathy Anderson, Ladue School District Julie Antill, Southeast RPDC, Cape Girardeau Jen Cooper, West Plains School District Terry Goodman, University of Central Missouri Tammy Hanebrink, Southeast Missouri State University Audrey Jackson, St. Louis School District Paula Jensen, Independence School District Julie Kubiak, Lee's Summit School District John Lannin, University of Missouri Kim Ramsey, Kansas City School District Carol Sipes, Kirkwood School District (retired) | <u>Grades 6-12 Writing Team</u> Cathy Battles, Chair, Lee's Summit School District (retired) Sherri Adams, Avila University Joann Barnett, Ozark School District Trish Goddard, Southwest RPDC, Springfield Curtis James, Clayton School District Mark Jarboe, Moberly School District Ann McCoy, University of Central Missouri Karen Pace, Salem School District Chip Sharp, Columbia School District James Tarr, University of Missouri Curtis Wren, Hazelwood School District |
|--|--|

Review Process

A Draft of the document was posted on the Missouri DESE website in late March 2008 and all interested individuals and groups were invited to provide feedback. In addition, state and national experts were individually invited to provide a review of the Draft. Over 300 pages of suggestions and comments were received. The Writing Group met in the summer of 2008 to review the feedback and to modify the Draft in response to feedback and to the recommendations included in the final report of the National Mathematics Advisory Panel report released in March 2008.

The organizing agencies (DESE and METS) and members of the Writing Group offer their sincere appreciation to everyone who offered feedback on the DRAFT. In particular, we note the following individuals who provided feedback at the request of DESE and METS:

Yungchen Cheng, mathematician, Missouri State University
Linda Coutts, district mathematics coordinator, Columbia Public Schools
Juli Dixon, member, Florida K-12 Mathematics Standards Writing Group, University of Central Florida
John Dossey, former President of the National Council of Teachers of Mathematics
Jim Fey, curriculum developer and researcher, University of Maryland (retired)
Karen Fuson, curriculum developer and researcher, Northwestern University (retired)
Mary Lindquist, former President of the National Council of Teachers of Mathematics
Vena Long, former DESE Mathematics Consultant, University of Tennessee
Richard Lodholz, mathematics coordinator, Parkway School District (retired)
Gary Martin, mathematics educator, Auburn University
John McCarthy, mathematician, Washington University
Jim Milgram, mathematician, Stanford University
Dorina Mitrea (with Tanya Christiansen), mathematician, University of Missouri
Janie Schelack, chair of the NCTM Curriculum Focal Points Writing Group, Texas A&M
Sue Sundberg, mathematics educator, University of Central Missouri
Marianne Weber, district mathematics coordinator, Webster Groves School District

The Writing Group commissioned by DESE and METS believes that every Missouri student should have access to a high-quality mathematics education. The mathematics content priorities, learning goals, and performance indicators outlined here are offered to advance that goal.

Cindy Bryant
Barbara Reys
Co-Chairs, K-12 Mathematics Learning Goals Writing Group

INTRODUCTION

The primary purpose of the *Missouri K-12 Mathematics Learning Goals* is to articulate a focused, coherent, and forward-looking mathematics program which provides basic preparation for careers and livelihood in today's technology- and statistics-rich environment for ALL Missouri students, even those who meet only the minimum graduation requirement in mathematics. For Missouri students graduating in 2010 and thereafter, 3 units of mathematics is required for high school graduation, with End-of Course exams required for Algebra I and one additional course selected from Geometry, Algebra II, Integrated Mathematics II or III. However, many students will need more mathematics preparation than is required or than is outlined here, notably:

- students preparing to attend college at many universities in and outside of Missouri will be expected to have completed four years of high school mathematics, including specified courses such as Algebra and Geometry, and
- students intending to pursue a mathematics-intensive career or college major should also study mathematics beyond that outlined here; in particular, they will need at least a fourth year of high school mathematics (e.g., Pre-Calculus or Integrated IV) and may also begin their study of calculus in high school. It is critical for the competitiveness of our state that we dramatically increase the amount of mathematics learned in grades K-12 for many more students.

Although the core content, learning goals, and performance indicators specified in this document are intended for ALL students, many Missouri students will be able to move through this content more quickly and will need more mathematics than is outlined here. For that reason, we urge local educational agencies to develop and implement policies and programs serving all students beginning in elementary school, including those who are ready for early advancement and need more mathematics than the material described in this document. As essential support for raising Missouri's performance in mathematics, specification of core content, learning goals, and performance indicators for fourth-year high school mathematics courses is under development.

Emphasis on Mathematical Content

The curriculum recommendations in this document emphasize the importance of *mathematics proficiency* (National Research Council, 2001) as the primary goal of mathematics instruction. Mathematics proficiency includes five interwoven and interdependent strands:

Conceptual understanding—comprehension of mathematical concepts, operations, and relations.

Procedural fluency—skill in carrying out procedures flexibly, accurately, efficiently, and appropriately.

Strategic competence—ability to formulate, represent, and solve mathematical problems.

Adaptive reasoning—capacity for logical thought, reflection, explanation, and justification.

Productive disposition—habitual inclination to see mathematics as sensible, useful, worthwhile, coupled with a belief in diligence and one's own efficacy. (NRC, 2001, p. 116)

The document is formatted differently from Grade-Level Expectations (GLE) documents previously published by DESE. Rather than a matrix format (organized by grade/course and strand), this document is organized by core content (content priorities) for each grade and course, followed by learning goals, and then by performance indicators.

Core Content – Important mathematical ideas/topics that serve as organizing structures for curriculum design, instruction, and assessment for a grade level or course.

Learning Goals - A set of statements, organized around each core content area, indicating what students are expected to learn.

Performance Indicators - Statements of specific and measurable learning outcomes.

The core content areas identified for each grade (K-8) and high school course represent a critical and unique element of this revision. They describe important mathematics to be learned and, along with the learning goals and performance indicators, build a cohesive curriculum K-12.

Performance Indicators have been reviewed to identify candidates for assessment at the state and local level. A performance indicator (PI) coded with an asterisk (*) is required for local, rather than state, assessment. A PI with no asterisk is a candidate for annual assessment at the state level (Grades 3 – 8 MAP Assessment or End-of-Course Exams). Locally assessed PIs are selected based on several considerations. For example, it may be that these topics are the focus of instruction at the end of the school year (following the state assessment calendar window). On the other hand, some of the locally assessed topics require special assessment considerations (e.g., access to special equipment) and therefore are not appropriate for state assessment.

Emphasis on Mathematical Processes

At every grade, students must be challenged to use mathematics to reason and solve problems and they must be provided opportunities to develop these mathematical processes. In addition, they must be able to communicate about mathematics including representing (in writing, speaking and various symbolic formats) mathematical ideas.

The National Council of Teachers of Mathematics (2000) describes five mathematical processes that are essential elements of K-12 mathematics. They include:

Problem Solving: Students learn to identify mathematical problems; to formulate and apply strategies to solve problems; and to convey solutions using appropriate terminology and mathematical representations.

Reasoning and Proof: Students learn to justify their thinking; to reinforce and extend logical reasoning abilities; to reflect on and clarify their thinking; and to ask questions to extend their thinking.

Communication: Students learn to solve problems by obtaining information through reading, listening, and observing; to translate this information into mathematical language and symbols; to process this information mathematically; and to present results in written, oral, and visual formats.

Connections: Students learn to relate various mathematical ideas and to use these relations to expand the ways they can approach problems.

Representations: Students learn to create and use representations to organize, record, and communicate mathematical ideas; to select, apply, and translate among mathematical representations to solve problems; to use representations to model and interpret physical, social, and mathematical phenomena.

In this document mathematical processes are interwoven throughout the core content and learning goals so that they are not thought of in isolation but rather as part of learning the content of mathematics. Key ideas related to mathematical processes are also listed in the “Core Content Matrix” (see pp. 8-9) by grade band.

Statistical Literacy

As in the previous GLEs, this document includes a strand devoted to the development of statistical literacy. However, it is not a focus of attention at every grade level. Rather, it is included as a core content area in some grades. The inclusion of this strand is based upon recommendations from various national groups of the increased importance of statistics as a basic knowledge for citizenry and work environments. The

American Statistical Association (2005) indicates:

Statistical literacy is essential in our personal lives as consumers, citizens, and professionals ... Sound statistical reasoning skills take a long time to develop. They cannot be honed to the level needed in the modern world through one high-school course. The surest way to help students attain the necessary skill level is to begin the statistics education process in the elementary grades and keep strengthening and expanding students' statistical thinking skills throughout the middle- and high-school years. (p. 3).

Lynn Steen, former President of the Mathematical Association of America, describes the need for statistical (quantitative) literacy in *Mathematics and Democracy: The Case for Quantitative Literacy* (2001):

Unfortunately, despite years of study and life experience in an environment immersed in data, many educated adults remain functionally innumerate...even individuals who have studied trigonometry and calculus often remain largely ignorant of common abuses of data and all too often find themselves unable to comprehend (much less to articulate) the nuances of quantitative inferences... Quantitative literacy empowers people by giving them tools to think for themselves, to ask intelligent questions of experts, and to confront authority confidently. These are the skills required to thrive in the modern world (pp. 1-2).

Statistical literacy is addressed in this document through a strand of core content that begins in the elementary grades and extends through the high school courses. While some high school students may also take a specific course on statistics, the strand outlined in this document provides a basis for all Missouri students to acquire basic statistical literacy.

Technology and Mathematics Teaching and Learning

Technology, used in appropriate ways, can increase opportunities for student learning, enrich learning environments, and prepare students for an increasingly technology-rich work and life environment. While some of the mathematics outlined in this document will be taught and learned without the use of technology, other mathematical topics can be learned and used with the technology. On Missouri state-mandated assessments, students in grades three through five are not permitted to use calculators during the exams. In grades six through eight, calculator use is limited; that is, calculators are not permitted on items that assess computational proficiency. In grades 6-8 and on End-of-Course assessments, students are permitted to use calculators, but are limited to using four-function, scientific, or graphing calculators. Students are prohibited from using calculators with built-in computer algebra systems.

The content priorities, learning goals, and performance indicators describe the mathematics students need to learn, not necessarily all of the tools that teachers might use to help students reach these goals. Teachers need to be knowledgeable about how technology can support students in learning mathematics, not replace their learning. More importantly, teachers need to know what mathematics students need to understand so that students can use the technology as well as make sense of the outputs from that technology.

Relationship to National Recommendations and Other DESE Documents

The *Missouri K-12 Mathematics Learning Goals* are built upon the foundation of the *Show-Me Content and Process Standards* (1996). Other important resources in the development of the document include:

- National Mathematics Advisory Panel (2008)
- National Assessment of Educational Progress framework (2007)
- National Council of Teachers of Mathematics *Principles and Standards for School Mathematics* (2000) and *Curriculum Focal Points* (2006)
- Achieve, Inc. *Elementary Mathematics Benchmarks, Grades K-6* (2008) and *Secondary Mathematics Benchmarks, Grades 7-12* (2008)
- American Statistical Association *Guidelines for Assessment and Instruction in Statistics Education* (2006)

- *College Board Standards for College Success (2007)*
- Missouri Department of Higher Education *Mathematics College Level Entrance Competencies (Draft, 2008)*.

In addition, documents with a similar purpose and design from key states (those whose students demonstrate consistently high performance on NAEP) were consulted and drawn from. These states include Minnesota, Maryland, Massachusetts, and Washington.

Using the Document

The document outlines 3-5 key “core content” areas for each grade/course. Each core content area is further elaborated with statements of “learning goals” and “performance indicators.” The document also includes an estimate of the intended relative emphasis of each core content area. That is, a percentage is noted for each core content area, ranging from 10% to 30%. The indicated percentages are estimates and should be considered in that light as local decisions are made. The sum of the percentages for any grade/course is 85% (except for Grade 8, where it is 90%), enabling individual schools or school districts to add additional content in response to local needs. For example, to reinforce the basic skills and concepts inherent in mathematical proficiency (defined above) and essential for further work in mathematics.

Mathematics is important and is also an inherently beautiful subject. Learning mathematics can and should be enjoyable, at times exciting, and personally rewarding. While these important attitudinal aspects of school mathematics are difficult to quantify in a document such as this, it is the responsibility of teachers to help students see and experience the joy of learning mathematics. In probably no other field is the relationship between a student’s doing well and a student’s liking of a subject more important than in mathematics.

Finally, it is fitting to close this introduction with a reminder of what has been noted earlier, namely, that the material specified here is for ALL students. But many students will be able to advance through this content more quickly than others and/or be able to study the mathematics outlined in more depth. In fact, it is common in many Missouri schools for some students in middle school to begin the formal study of algebra as outlined in the high school section of this document. Local educational agencies are urged to keep in mind the needs of these students and make provision for the modification of the outlined material for students advancing at an accelerated pace.

REFERENCES AND PRIMARY RESOURCES

- Achieve. (2007). *Secondary mathematics benchmarks, Grades 7-12*. Retrieved July 28, 2008, from <http://www.achieve.org/node/966>.
- Achieve. (2007). *Elementary mathematics benchmarks, Grades K-6*. Retrieved July 28, 2008, from <http://www.achieve.org/node/968>.
- American Statistical Association. (2005). *Guidelines for assessment and instruction in statistics education (GAISE) report*. Retrieved July 28, 2008 from <http://www.amstat.org/education/gaise/>
- Carpenter, Thomas P., Elizabeth Fennema, Megan Loef Franke, Linda Levi, and Susan B. Empson. (1999). *Children's Mathematics: Cognitively Guided Instruction*. Portsmouth, NH: Heinemann.
- Carpenter, Thomas P., Megan Loef Franke, and Linda Levi. (2003). *Thinking Mathematically: Integrating Arithmetic and Algebra in Elementary School*. Portsmouth, NH: Heinemann.
- College Board. (2006). *College Board standards for college success: Mathematics and statistics*. Retrieved March 6, 2008, from <http://professionals.collegeboard.com/k-12/system/standards>
- College Board. (2006). *College Board standards for college success: Mathematics and statistics* (Adapted for integrated curricula). Retrieved March 6, 2008, from <http://professionals.collegeboard.com/k-12/system/standards>
- College Board. (2006). *College Board standards for college success: Mathematics and statistics* (Three year alternative for middle school). Retrieved March 6, 2008, from <http://professionals.collegeboard.com/k-12/system/standards>
- Missouri Department of Elementary and Secondary Education. (1996). *Show-Me Standards*. Retrieved March 6, 2008 from <http://dese.mo.gov/standards/>
- Missouri Department of Elementary and Secondary Education. (1996). *Missouri's frameworks for curriculum development: Mathematics*. Retrieved March 6, 2008 from <http://dese.mo.gov/divimprove/curriculum/frameworks/math.html>
- Missouri Department of Elementary and Secondary Education. (2008). *Mathematics grade- and course-level expectations*. Retrieved March 6, 2008 from <http://dese.mo.gov/divimprove/curriculum/GLE/index.html>
- Missouri Department of Higher Education. (2008). *Mathematics college level entry competencies, Mathematics (DRAFT)*. Retrieved August 14, 2008, from <http://www.dhe.mo.gov/files/MathematicsEntryLevelCompetencies.pdf>
- Mathematics Engineering Technology & Science (METS) Alliance Report. (2006). Retrieved March 6, 2008, from <http://www.missourimets.com/docs/METS%20Alliance%20Report.pdf>
- National Assessment of Educational Progress. (2007). *Mathematics framework for 2009*, Pre-Publication Edition. Retrieved July 28, 2008 from <http://www.nagb.org/>.
- National Council on Education and the Disciplines. (2001). *Mathematics and democracy: The Case for quantitative literacy*. Retrieved July 28, 2008 from <http://www.maa.org/ql/mathanddemocracy.html>
- National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: The Council.
- National Council of Teachers of Mathematics. (2006). *Curriculum focal points for prekindergarten through grade 8 mathematics*. Reston, VA: The Council.

National Mathematics Advisory Panel. 2008. *Foundations for success*. Washington, D.C.: U.S. Department of Education.

National Research Council. (2001). *Adding it up: Helping children learn mathematics*. Washington, D.C.: The National Academies Press.

Schifter, Deborah, Virginia Bastable, and Susan Jo Russell with Kristine Reed Woleck. (2002). *Developing mathematical ideas: Measuring space in one, two, and three dimensions (Casebook)*. Parsippany, NJ: Dale Seymour Publications.

U.S. Department of Education. (2008). The Final report of the National Mathematics Advisory Panel. Retrieved March 14, 2008 from <http://www.ed.gov/about/bdscomm/list/mathpanel/index.html>

Van de Walle, John A., and LouAnn H. Lovin. (2006). *Teaching student-centered mathematics: Grades 3-5*. Boston: Pearson Education, Inc.

Webb, Norm. (1998). *Depth-of-knowledge levels definitions*. Retrieved March 6, 2008, <http://facstaff.wcer.wisc.edu/normw/state%20alignment%20page%20one.htm>

Key other state documents utilized in the preparation of this document:

Maryland Voluntary State Curriculum (2004) –
<http://www.mdk12.org/instruction/curriculum/index.html>

Massachusetts Mathematics Curriculum Framework (2000) -
<http://www.doe.mass.edu/frameworks/current.html>

Minnesota Academic Standards for Mathematics K to 12 (2007) -
http://education.state.mn.us/MDE/Academic_Excellence/Academic_Standards/Mathematics/index.html

Washington K-12 Mathematics Standards (2008) -
<http://www.k12.wa.us/CurriculumInstruct/Mathematics/default.aspx>

**Summary of Mathematics Core Content:
Kindergarten through High School**

DRAFT

Summary of Mathematics Core Content: Grades K-8

| | Number/Algebra | Geometry/Measurement | Data/Probability | Problem Solving/Reasoning Communication/Connections/ Representation |
|-------|---|--|--|--|
| Gr. K | Counting (30%) Beginning Addition and Subtraction (25%) | Geometric Shapes and Spatial Relationships (20%) Informal Measurement (10%) | | Use a variety of strategies to solve problems. Organize, record, and describe in words mathematical thinking. |
| Gr. 1 | Whole Number Relationships (25%) Addition and Subtraction: Operations and Beginning Basic Facts (40%) | Geometric Relationships (20%) | | |
| Gr. 2 | Base-ten Numeration and Place Value (20%) Addition and Subtraction: Basic Facts Fluency and Multi-digit Computation (35%) | Linear Measurement (20%) | Categorical Data (10%) | |
| Gr. 3 | Extending Addition and Subtraction (15%) Multiplication and Division: Operations and Beginning Basic Facts (25%) Meanings of Fractions and Fractional Relationships (25%) | Properties and Perimeter of Two-dimensional Shapes (20%) | | Apply and adapt a variety of strategies to solve problems. Make and investigate mathematical conjectures. |
| Gr. 4 | Multiplication and Division: Basic Facts Fluency and Multi-digit Computation (30%) Decimal Place Value and Connections to Decimals and Percents (30%) | Area Measurement and Transformations (15%) | Data Organization and Analysis (10%) | Communicate mathematical thinking coherently and clearly to peers and teacher. Organize, record, communicate, and represent mathematical ideas. |
| Gr. 5 | Division of Whole Numbers (30%) Addition and Subtraction of Fractions and Decimals (30%) | Properties of Three-dimensional Shapes, Volume and Surface Area (15%) | Basic Probability Concepts (10%) | |
| Gr. 6 | Multiplication and Division of Fractions and Decimals (10%) Ratios, Rates, and Percents (15%) Mathematical Expressions and Equations (20%) | Extending Properties and Measures of Two-dimensional Shapes (25%) | Sample Surveys and Data Distributions (15%) | Apply and adapt a variety of strategies to solve problems. Select and use various types of reasoning and methods of proof. |
| Gr. 7 | Rational Numbers and Linear Equations (30%) Proportionality and Similarity (25%) | Surface Area and Volume (20%) | Experimental and Theoretical Probability (10%) | Communicate mathematical thinking coherently and clearly to peers, teachers, and others. Use the language of mathematics to express mathematical ideas precisely. |
| Gr. 8 | Real Numbers, Polynomial Expressions and Operations on Polynomials Expressions (15%) Linear Functions and Equations (25%) | Transformation (15%) Distance and Angle Measurement (20%) | Bivariate Data (15%) | Create, select, apply, and translate among mathematical representations to solve problems. |

Note: A percentage of instructional time for each core content area is suggested. The sum of the percentages totals less than 100% so that school districts can modify the emphasis and/or add additional content in response to local needs.

Summary of Mathematics Core Content: High School

| Course | Number/Algebra | Geometry/Masurement | Data/Probability | Problem Solving/Reasoning Communication/Connections/ Representation |
|----------------|---|--|--|---|
| Algebra I | Expressions, Linear Equations, and Inequalities (30%) Functions (20%) Exponential, Quadratic and Other Non-Linear Equations (25%) | | Trend Lines and Correlation (10%) | Solve problems by applying, generating, and/or adapting mathematical tools and strategies. Make and investigate mathematical conjectures. Develop and evaluate mathematical arguments and proofs. |
| Geometry | | Geometric Reasoning, Proof, and Representation (25%) Similarity and Congruency (25%) Direct and Indirect Measurement (25%) | Conditional Probability and Independence (10%) | Use the language of mathematics to express mathematical ideas precisely. Understand how mathematical ideas interconnect. |
| Algebra II | Polynomial Expressions, Equations, and Functions (30%) Exponential, Logarithmic and Other Functions, and Equations (30%) Systems of Equations and Inequalities and Matrices (15%) | | Binomial Theorem and Probability (10%) | Create and use representations to organize, record, and communicate mathematical ideas. Select, apply, and translate among mathematical representations to solve problems and model situations. |
| Integrated I | Expressions, Linear Equations, and Inequalities (25%) Functions (15%) Nonlinear Functions (20%) | Geometric Reasoning and Relationships (15%) | Trend Lines and Correlation (10%) | Solve problems by applying, generating, and/or adapting mathematical tools and strategies. Make and investigate mathematical conjectures. Develop and evaluate mathematical arguments and proofs. |
| Integrated II | Systems of Linear Equations and Matrices (25%) | Geometric Proof, Similarity, and Transformations (30%) Direct and Indirect Measurement (20%) | Conditional Probability and Independence (10%) | Use the language of mathematics to express mathematical ideas precisely. Understand how mathematical ideas interconnect. |
| Integrated III | Polynomial Expressions, Equations, and Functions (30%) Exponential, Logarithmic and Other Functions, and Equations (25%) Sequences and Recursion (10%) | Indirect Measurement (10%) | Binomial Theorem and Probability (10%) | Create and use representations to organize, record, and communicate mathematical ideas. Select, apply, and translate among mathematical representations to solve problems and model situations. |

Note: A percentage of instructional time for each core content area is suggested. The sum of the percentages totals less than 100% so that school districts can modify the emphasis and/or add additional content in response to local needs.

DRAFT

Core Content, Learning Goals, and Performance Indicators

Grades K-5

DRAFT

KINDERGARTEN Summary of Core Content

A percentage of instructional time for each core content area is suggested. The sum of the percentages totals less than 100% so that school districts can modify the emphasis and/or add additional content in response to local needs.

Core Content A: Counting

30%

Students develop an understanding of whole numbers, number patterns and relationships, and the concepts underlying counting. They recognize the number of objects in small groups with and without counting. They understand that number words refer to quantity, use one-to-one correspondence, and produce sets of given amounts. They use patterns with objects, sounds, and movements as a foundation for numeric relationships.

Core Content B: Beginning Addition and Subtraction

25%

Students develop an understanding of the relationships among whole numbers and of joining and separating sets. They use numbers, including written numerals, to solve quantitative problems. They compare and order sets or numerals and understand that numbers occurring later in a number sequence are larger than those at the beginning of the number sequence. They model simple joining and separating situations with objects. They choose, combine, and apply effective strategies for answering quantitative questions, including comparing and ordering sets or numerals and counting the number in individual or combined sets.

Core Content C: Geometric Shapes and Spatial Relationships

20%

Students develop an awareness of geometric shapes and spatial relationships. They interpret the physical world with geometric ideas (e.g., shape, orientation, spatial relations) and describe it with corresponding vocabulary. They identify, name, and describe a variety of two- and three-dimensional shapes presented in a variety of ways (e.g., different sizes or orientations). They use basic shapes and spatial reasoning to model objects in their environment.

Core Content D: Informal Measurement

10%

Students develop an understanding of measurable attributes and use informal processes of measurement to compare and order objects. They compare the lengths of two objects both directly (by comparing them with each other) and indirectly (by comparing both with a third object), and they order several objects according to length.

NOTE: The order of the Core Content, Learning Goals, and Performance Indicators on the pages that follow does not prescribe a sequence of instruction. Sequencing should be determined by individual school districts based on their local curricula and instructional programs.

NOTE: No state assessments are administered in grades K – 2, therefore all Performance Indicators in kindergarten are locally assessed.

KINDERGARTEN

Kindergarten, Core Content A: Counting

30%

Students develop an understanding of whole numbers, number patterns and relationships, and the concepts underlying counting. They recognize the number of objects in small groups with and without counting. They understand that number words refer to quantity, use one-to-one correspondence, and produce sets of given amounts. They use patterns with objects, sounds, and movements as a foundation for numeric relationships.

Learning Goals and Performance (*Locally Assessed) Indicators:

1. Understand and apply the counting process to various situations.
 - a) *Count forward at least to 31 and backward from 10.
 - b) *Use one-to-one correspondence when counting a set of objects and keep track of counted and uncounted objects.
 - c) *Know that the last counting word represents the number in the collection.
 - d) *Produce sets of given amounts (e.g., show 6 objects).
 - e) *Recognize that the rearrangement of objects in a set does not change the number of objects.
2. Understand and use patterns as a foundation for describing relationships among whole numbers.
 - a) *Describe, extend, and create sequential patterns (objects, sounds, movements, and numbers) based on repeating terms and recognize that some patterns may have many possible repeating terms.
 - b) *Apply the 0-9 sequence when counting or writing numerals from one decade to the next decade.
 - c) *Recognize that a teen number has one group of ten plus some ones.

Kindergarten, Core Content B: Beginning Addition and Subtraction**25%**

Students develop an understanding of the relationships among whole numbers and of joining and separating sets. They use numbers, including written numerals, to solve quantitative problems. They compare and order sets or numerals and understand that numbers occurring later in a number sequence are larger than those at the beginning of the number sequence. They model simple joining and separating situations with objects. They choose, combine, and apply effective strategies for answering quantitative questions, including comparing and ordering sets or numerals and counting the number in individual or combined sets.

Learning Goals and Performance (*Locally Assessed) Indicators:

1. Understand the relative magnitude of numbers at least to 31.
 - a) *Represent numbers in a variety of ways and make connections between and among these representations by using objects, diagrams, words, and numerals.
 - b) *Order numerals and recognize that numbers occurring later in a number sequence are larger than those at the beginning of the number sequence.
 - c) *Compare sets of objects and determine whether they have the same, fewer, or more objects by matching and by counting.
 - d) *Solve problems that involve comparing and ordering quantities.
2. Understand the meaning of addition and subtraction for whole numbers less than ten.
 - a) *Represent and solve problems involving the joining and separating of sets by using objects, diagrams, words, and numerals.
 - b) *Compose and decompose quantities in order to establish relationships between the parts and the whole and identify all pairs of whole numbers that make a given number (e.g., 0 and 5; 1 and 4; 2 and 3 all make 5).

Kindergarten, Core Content C: Geometric Shapes and Spatial Relationships**20%**

Students develop an awareness of geometric shapes and spatial relationships. They interpret the physical world with geometric ideas (e.g., shape, orientation, spatial relations) and describe it with corresponding vocabulary. They identify, name, and describe a variety of two- and three-dimensional shapes presented in a variety of ways (e.g., different sizes or orientations). They use basic shapes and spatial reasoning to model objects in their environment.

Learning Goals and Performance (*Locally Assessed) Indicators:

1. Understand that two-dimensional and three-dimensional shapes can be classified on the basis of their attributes.
 - a) *Identify, name, and describe two-dimensional shapes, including circles, triangles, rectangles, and squares.
 - b) *Identify, name, and describe three-dimensional shapes, including pyramids, rectangular prisms, cubes, cones, cylinders, and spheres.
 - c) *Identify and describe shapes represented by physical models in the environment.
 - d) *Sort objects into groups by attribute (shape or size) and identify which attribute was used.
2. Understand relative positions of objects in space.
 - a) *Recognize that when shapes are moved (e.g., turned or shifted), they maintain the same classification.
 - b) *Demonstrate relative positions in space (*over, under, above, below, on, beside, next to, and between*).

Kindergarten, Core Content D: Informal Measurement**10%**

Students develop an understanding of measurable attributes and use informal processes of measurement to compare and order objects. They compare the lengths of two objects both directly (by comparing them with each other) and indirectly (by comparing both with a third object), and they order several objects according to length.

Learning Goals and Performance (*Locally Assessed) Indicators:

1. Understand measurable attributes of objects through informal measurement.
 - a) *Order objects directly or indirectly by using the measurable attributes of length.
 - b) *Compare and describe the length of a group of objects (e.g., longer, longest, taller, tallest, shorter, shortest).

GRADE 1

A percentage of instructional time for each core content area is suggested. The sum of the percentages totals less than 100% so that school districts can modify the emphasis and/or add additional content in response to local needs.

Grade 1, Core Content A: Whole Number Relationships

25%

Students develop an understanding of whole number relationships, including grouping in tens and ones. They compare and order whole numbers to develop an understanding of and solve problems involving the relative magnitude of these numbers. They think of whole numbers between 10 and 100 in groups of tens and ones (especially recognizing the numbers 11 to 19 as 1 group of ten and particular numbers of ones). They understand the sequential order of the counting numbers and represent numbers in a variety of ways.

Grade 1, Core Content B: Addition and Subtraction: Operations and Beginning Basic Facts

40%

Students develop an understanding of the operations of addition and subtraction, apply a variety of strategies for basic addition/subtraction facts, and relate addition and subtraction as inverse operations. They use a variety of models to develop an understanding of the meanings of addition and subtraction and strategies to solve arithmetic problems. They understand the connections between counting up and counting back with the operations of addition and subtraction (e.g., adding two is the same as “counting on” two). They use properties of addition (identity, commutative, and associative) to add whole numbers, and they create and use increasingly sophisticated strategies based on these properties to solve addition and subtraction problems involving basic facts. By comparing a variety of solution strategies, children relate addition and subtraction as inverse operations.

Grade 1, Core Content C: Geometric Relationships

20%

Students develop an understanding of the relationships among geometric shapes. They compose and decompose plane and solid figures (e.g., by putting two congruent isosceles triangles together to make a rhombus), thus building an understanding of part-whole relationships as well as the properties of the original and composite shapes. By combining figures, they recognize them from different perspectives and orientations, describe their geometric attributes and properties, and determine how they are alike and different. Through these processes, they develop a background for measurement (e.g., area and volume) and initial understandings of such properties as congruence and symmetry.

NOTE: The order of the Core Content, Learning Goals, and Performance Indicators on the pages that follow does not prescribe a sequence of instruction. Sequencing should be determined by individual school districts based on their local curricula and instructional programs.

NOTE: No state assessments are administered in grades K – 2, therefore all Performance Indicators in grade 1 are locally assessed.

Grade 1, Core Content A: Whole Number Relationships**25%**

Students develop an understanding of whole number relationships, including grouping in tens and ones. They compare and order whole numbers to develop an understanding of and solve problems involving the relative magnitude of these numbers. They think of whole numbers between 10 and 100 in groups of tens and ones (especially recognizing the numbers 11 to 19 as 1 group of ten and particular numbers of ones). They understand the sequential order of the counting numbers and represent numbers in a variety of ways.

Learning Goals and Performance (*Locally Assessed) Indicators:

1. Understand and explain the relative magnitude of numbers to 100.
 - a) *Represent two-digit numbers in a variety of ways and make connections between and among these representations using objects, diagrams, number lines, words, and numerals.
 - b) *Read, write, compare, and order numbers through 100 by using decades as benchmarks (e.g., 27 is between 20 and 30) and representations including the number line.
 - c) *Group and count objects by 2s, 5s, and 10s.
 - d) *Identify numbers missing from a well-defined counting sequence (e.g., 0, 2, 4, __, 8, 10, ...)
2. Understand the values of the digits in two-digit numbers.
 - a) *Determine the place value (tens, ones) and value of each digit in a number (e.g., the 6 in 63 represents 6 tens or 60 ones).
 - b) *Translate between and among different numerical representations of a number (e.g., 63 is 6 tens plus 3 ones or 5 tens plus 13 ones or 4 tens plus 23 ones).
 - c) *Identify one more, one less, 10 more, and 10 less than a given number for numbers up to 100.
3. Understand the properties of odd and even numbers.
 - a) *Classify a number as odd or even and explain why it is odd or even.

Grade 1, Core Content B: Addition and Subtraction: Operations and Beginning Basic Facts 40%

Students develop an understanding of the operations of addition and subtraction, apply a variety of strategies for basic addition/subtraction facts, and relate addition and subtraction as inverse operations. They use a variety of models to develop an understanding of the meanings of addition and subtraction and strategies to solve arithmetic problems. They understand the connections between counting up and counting back with the operations of addition and subtraction (e.g., adding two is the same as “counting on” two). They use properties of addition (identity, commutative, and associative) to add whole numbers, and they create and use increasingly sophisticated strategies based on these properties to solve addition and subtraction problems involving basic facts. By comparing a variety of solution strategies, children relate addition and subtraction as inverse operations.

Learning Goals and Performance (*Locally Assessed) Indicators:

1. Understand and explain the meaning of addition and subtraction.
 - a) *Identify addition and subtraction situations using interpretations such as combining (addition) and missing addend (subtraction).
 - b) *Compute addition facts (one-digit addends) and related subtraction facts by using strategies based on properties of operations (identity, commutative, associative) and/or the inverse relationship between addition and subtraction.
2. Use a variety of strategies to represent and compute single-digit addition and related subtraction facts.
 - a) *Interpret the equal sign as a relational symbol indicating “the same quantity as.”
 - b) *Use informal strategies to find the unknown quantity in a variety of number sentences that involve addition and subtraction (e.g., $3 + 4 = \underline{\quad}$; $7 - \underline{\quad} = 3$; $\underline{\quad} = 5 - 2$).
 - c) *Compare solution strategies in order to relate addition and subtraction as inverse operations (e.g., missing addend situations).
 - d) *Apply and justify the use of a variety of strategies, including known facts and derived facts (e.g., counting on or counting back, doubling plus one, doubling minus one, making ten), to solve problems.
 - e) *Create problems for a variety of mathematical situations (combining, missing addend, separating, comparing, and relating parts with wholes) involving numbers for which any one of the quantities is unknown.
 - f) *Solve problems involving a variety of mathematical situations (combining, missing addend, separating, comparing, and relating parts with wholes) for which any one of the quantities is unknown.

Grade 1, Core Content C: Geometric Relationships**20%**

Students develop an understanding of the relationships among geometric shapes. They compose and decompose plane and solid figures (e.g., by putting two congruent isosceles triangles together to make a rhombus), thus building an understanding of part-whole relationships as well as the properties of the original and composite shapes. By combining figures, they recognize them from different perspectives and orientations, describe their geometric attributes and properties, and determine how they are alike and different. Through these processes, they develop a background for measurement (e.g., area and volume) and initial understandings of such properties as congruence and symmetry.

Learning Goals and Performance (*Locally Assessed) Indicators:

1. Understand part-whole relationships and attributes of plane and solid figures.
 - a) *Name, create, and sort 2-dimensional shapes, including circles, triangles, rectangles, squares, rhombi, trapezoids, and hexagons.
 - b) *Sort 3-dimensional shapes including pyramids, rectangular prisms, cubes, cones, cylinders, and spheres.
 - c) *Compose (combine) and decompose (separate) two- and three-dimensional shapes to make other shapes.
 - d) *Predict the number of congruent shapes that will cover a region without gaps, then test and verify predictions with models.
 - e) *Identify two-dimensional shapes that have line (mirror) symmetry and draw line(s) of symmetry.

GRADE 2

A percentage of instructional time for each core content area is suggested. The sum of the percentages totals less than 100% so that school districts can modify the emphasis and/or add additional content in response to local needs.

Grade 2, Core Content A: Base-ten Numeration and Place Value

20%

Students develop an understanding of the base-ten numeration system and place-value concepts. They count in units and multiples of hundreds. They compare and order numbers. They understand numbers in terms of place value, recognizing that place-value notation is shorthand for the sums of multiples of powers of 10 (e.g., 853 as 8 hundreds + 5 tens + 3 ones; $800 + 50 + 3$).

Grade 2, Core Content B: Addition and Subtraction: Basic Facts Fluency and Multi-digit Computation

35%

Students demonstrate proficiency with basic facts for addition and subtraction and develop strategies for adding and subtracting multi-digit numbers. They solve problems by applying their understanding of models of addition and subtraction (such as combining or separating sets or using number lines), relationships and properties of number (such as place value), and properties of addition. They apply appropriate methods to estimate sums and differences or calculate them mentally, depending on the context and numbers involved. They demonstrate proficiency (accuracy, efficiency, flexibility, and appropriateness) with computational procedures, including a standard algorithm, for adding and subtracting whole numbers (with sums at least to 1000 and minuends at least to 100). They understand and explain why computational procedures work (on the basis of place value and properties of operations), and use them to solve problems.

Grade 2, Core Content C: Linear Measurement

20%

Students develop an understanding of linear measurement and facility in measuring lengths by examining and quantifying objects in the real-world. They understand underlying concepts of measurement, such as equal partitioning (the mental activity of slicing the length of an object into equal-sized units) and transitivity (e.g., if object A is longer than object B and object B is longer than object C, then object A is longer than object C). They understand that measurements are approximations and that measuring tools (e.g., ruler) provide a means of measuring objects to the nearest appropriate unit (e.g., nearest inch). They understand linear measure as an iteration of units (repetition of a single unit with no gaps or overlaps) and use non-standard measurement tools and simple unit rulers (centimeter and inch) on that basis. They understand the need for equal-length units and recognize the inverse relationship between the size of a unit and the number of units used in a particular measurement (e.g., the smaller the unit, the more iterations needed to cover a given length).

Grade 2, Core Content D: Categorical Data

10%

Students develop an initial understanding of data analysis by formulating questions and simple experiments and by collecting, representing, analyzing, and interpreting data. They pose and investigate questions that can be addressed with categorical data about a small population. They conduct classroom censuses and use a variety of displays to represent and interpret data.

NOTE: The order of the Core Content, Learning Goals, and Performance Indicators on the pages that follow does not prescribe a sequence of instruction. Sequencing should be determined by individual school districts based on their local curricula and instructional programs.

NOTE: No state assessments are administered in grades K – 2, therefore all Performance Indicators in Grade 2 are locally assessed.

Grade 2, Core Content A: Base-ten Numeration and Place Value**20%**

Students develop an understanding of the base-ten numeration system and place-value concepts. They count in units and multiples of hundreds. They compare and order numbers. They understand numbers in terms of place value, recognizing that place-value notation is shorthand for the sums of multiples of powers of 10 (e.g., 853 as 8 hundreds + 5 tens + 3 ones; $800 + 50 + 3$).

Learning Goals and Performance (*Locally Assessed) Indicators:

1. Understand and explain the relative magnitude of three-digit numbers.
 - a) *Use a variety of representations to model and compare whole numbers up to 999 (e.g., base ten blocks, numeric forms and number lines).
 - b) *Read, write, compare, and order numbers through 1000 by using decades and hundreds as benchmarks (e.g., 270 is between 200 and 300).

2. Understand and explain the values of the digits in three-digit numbers.
 - a) *Recognize the place value (hundreds, tens, ones) and value of each digit in a number (e.g., the 6 in 678 represents 6 hundreds, 60 tens, or 600 ones).
 - b) *Translate among different numerical representations of a number (e.g., 678 is 6 hundreds plus 7 tens plus 8 ones or 6 hundreds plus 6 tens plus 18 ones, or 4 hundreds plus 27 tens plus 8 ones).
 - c) *Identify 10 more or 10 less and 100 more or 100 less than a given number for numbers to at least 1000.

Grade 2, Core Content B: Addition and Subtraction: Basic Facts Fluency and Multi-digit Computation

35%

Students demonstrate proficiency with basic facts for addition and subtraction and develop strategies for adding and subtracting multi-digit numbers. They solve problems by applying their understanding of models of addition and subtraction (such as combining or separating sets or using number lines), relationships and properties of number (such as place value), and properties of addition. They apply appropriate methods to estimate sums and differences or calculate them mentally, depending on the context and numbers involved. They demonstrate proficiency (accuracy, efficiency, flexibility, and appropriateness) with computational procedures, including a standard algorithm, for adding and subtracting whole numbers (with sums at least to 1000 and minuends at least to 100). They understand and explain why computational procedures work (on the basis of place value and properties of operations), and use them to solve problems.

Learning Goals and Performance (*Locally Assessed) Indicators:

1. Understand and use efficient strategies for computing single-digit addition and subtraction facts.
 - a) *Demonstrate proficiency computing addition and subtraction facts (single-digit addends and related subtraction facts).
 - b) *Represent addition and subtraction situations by using objects, diagrams, words, expressions, and equations and make connections among the representations.
 - c) *Explain and justify addition and subtraction strategies on the basis of place value concepts, properties of operations (identity, commutative, associative) and/or the inverse relationship between addition and subtraction.
2. Understand, explain, and use a variety of strategies to proficiently compute multi-digit addition and subtraction problems (sums to 1000 and minuends to 100).
 - a) *Analyze a variety of strategies (including a standard algorithm) for addition and subtraction in order to demonstrate their similarities and differences, and to draw conclusions about their efficiency, accuracy, and generalizability.
 - b) *Demonstrate proficiency in adding and subtracting multi-digit numbers (sums to 1000 and minuends to 100).
 - c) *Estimate sums and differences and/or calculate them mentally depending on the context and numbers involved; use estimates to determine the reasonableness of solutions.
 - d) *Recognize and apply the meaning of relational signs ($=$, \neq , $<$, $>$) as distinct from operational signs ($+$, $-$) using these symbols in number sentences (e.g., $3 + 4 = 7$; $3 + 4 < 8$).
 - e) *Find the unknown quantity in simple number sentences that involve addition and subtraction (e.g., $6 + 4 = \underline{\quad}$; $8 = 14 - \underline{\quad}$; $8 + 4 = \underline{\quad} + 2$; $\underline{\quad} = 15 - 2$).
 - f) *Create problems for a variety of mathematical situations (combining, missing addend, separating, comparing, and relating parts with wholes) involving numbers for which any one of the quantities is unknown.
 - g) *Solve problems for a variety of mathematical situations (combining, missing addend, separating, comparing, and relating parts with wholes) involving numbers for which any one of the quantities is unknown.

Grade 2, Core Content C: Linear Measurement**20%**

Students develop an understanding of linear measurement and facility in measuring lengths by examining and quantifying objects in the real-world. They understand underlying concepts of measurement, such as equal partitioning (the mental activity of slicing the length of an object into equal-sized units) and transitivity (e.g., if object A is longer than object B and object B is longer than object C, then object A is longer than object C). They understand that measurements are approximations and that measuring tools (e.g., ruler) provide a means of measuring objects to the nearest appropriate unit (e.g., nearest inch). They understand linear measure as an iteration of units (repetition of a single unit with no gaps or overlaps) and use non-standard measurement tools and simple unit rulers (centimeter and inch) on that basis. They understand the need for equal-length units and recognize the inverse relationship between the size of a unit and the number of units used in a particular measurement (e.g., the smaller the unit, the more iterations needed to cover a given length).

Learning Goals and Performance (*Locally Assessed) Indicators:

1. Understand and explain the measurement process (choosing a unit, comparing the unit to the object, and determining the number of units needed to represent the object) and apply it to measure real-world objects.
 - a) *Choose and use nonstandard and standard units (customary and metric) to measure length to the nearest whole unit.
 - b) *Estimate length by using nonstandard units and standard units (inches and centimeters).
 - c) *Use many copies of the same unit or iterate a single unit to measure the length of an object longer than the unit.
 - d) *Recognize that when multiple identical units are used for measurement, they must match up with the beginning of the object being measured and there can be no gaps or overlaps between units.
 - e) *Use direct comparison and measurement, along with concepts of the transitive property ($a > b$, $b > c$, so $a > c$) when comparing and ordering objects by the attribute of length.
 - f) *Describe the inverse relationship between the size of a unit and the number of units to measure an object.
 - g) *Recognize each unit on a ruler as completing a linear distance (length) from the zero point, not a location on the tool.

Grade 2, Core Content D: Categorical Data**10%**

Students develop an initial understanding of data analysis by formulating questions and simple experiments and by collecting, representing, analyzing, and interpreting data. They pose and investigate questions that can be addressed with categorical data about a small population. They conduct classroom censuses and use a variety of displays to represent and interpret data.

Learning Goals and Performance (*Locally Assessed) Indicators:

1. Understand and explain how to collect, represent, and interpret categorical data in response to questions posed by the class.
 - a) *Collect categorical data by conducting classroom censuses based on questions posed by the class.
 - b) *Use one-to-one correspondence in constructing frequency tables (with tallies or numbers), bar graphs, and picture graphs in order to represent and interpret the data.
 - c) *Recognize different representations of the same data and attend to the ideas of distribution and variation within the data set.
 - d) *Determine the mode (i.e., the most frequent data point) from a categorical data set (e.g., more students wearing “tennis shoes” than “flip-flops”).
 - e) *Compare responses (individual-to-individual and individual-to-group) and acknowledge that results may be different in another class or group.

GRADE 3

A percentage of instructional time for each core content area is suggested. The sum of the percentages totals less than 100% so that school districts can modify the emphasis and/or add additional content in response to local needs.

Grade 3, Core Content A: Extending Addition and Subtraction

15%

Students consolidate addition and subtraction strategies for larger numbers. They develop proficiency (accuracy, efficiency, flexibility, and appropriateness) with addition and subtraction of larger numbers, including standard algorithms, understand why these procedures work, and use them to solve problems.

Grade 3, Core Content B: Multiplication and Division: Operations and Beginning Basic Facts 25%

Students develop an initial understanding of multiplication and division, relate multiplication and division as inverse operations, and apply a variety of strategies for multiplication and division basic facts. They understand the meanings of multiplication and division of whole numbers through the use of various representations and situations (equal groups, arrays and area, multiplicative comparisons). They use properties of addition and multiplication to multiply whole numbers and apply increasingly sophisticated strategies based on these properties to solve multiplication and division problems involving basic facts (single-digit factors and related division facts). By comparing a variety of solution strategies, students relate multiplication and division as inverse operations.

Grade 3, Core Content C: Meanings of Fractions and Fractional Relationships

25%

Students develop an understanding of fractions and fraction equivalence by using area, length, and set models. They understand that the size of a fractional part is relative to the size of the whole, and they use fractions to represent numbers that are equal to, less than, or greater than 1. They solve problems that involve comparing and ordering fractions by using models, benchmark fractions, number lines or common numerators or denominators. They understand and use models to identify equivalent fractions.

Grade 3, Core Content D: Properties and Perimeter of Two-dimensional Shapes

20%

Students develop an understanding of properties and relationships in two-dimensional space and apply these ideas to measure real-world objects. They use benchmarks for classifying angles. They describe, analyze, compare, and classify two-dimensional shapes by their sides and angles and connect these attributes to the definitions of shapes. Students investigate, describe, and reason about composing and decomposing polygons to make other polygons. Through building, drawing, and analyzing two-dimensional shapes, students understand attributes and properties of two-dimensional space. They use these attributes and properties in solving problems, including applications involving perimeter.

NOTE: The order of the Core Content, Learning Goals, and Performance Indicators on the pages that follow does not prescribe a sequence of instruction. Sequencing should be determined by individual school districts based on their local curricula and instructional programs.

Grade 3, Core Content A: Extending Addition and Subtraction**15%**

Students consolidate addition and subtraction strategies for larger numbers. They develop proficiency (accuracy, efficiency, flexibility, and appropriateness) with addition and subtraction of larger numbers, including standard algorithms, understand why these procedures work, and use them to solve problems.

Learning Goals and Performance (*Locally Assessed) Indicators:

1. Understand, explain, and use a variety of strategies to compute multi-digit addition and subtraction problems proficiently (sums larger than 1000 and minuends larger than 100).
 - a) Demonstrate proficiency adding and subtracting multi-digit whole numbers.
 - b) *Estimate sums and differences and/or calculate them mentally depending on the situation and numbers involved; use estimates to judge the reasonableness of solutions.
 - c) *Create single- and multi-step problems for a variety of mathematical situations (combining, missing addend, separating, comparing, and relating parts with wholes) involving multi-digit numbers.
 - d) Solve single- and multi-step problems for a variety of mathematical situations (combining, missing addend, separating, comparing, and relating parts with wholes) involving multi-digit numbers.

Grade 3, Core Content B: Multiplication and Division: Operations and Beginning Basic Facts 25%

Students develop an initial understanding of multiplication and division, relate multiplication and division as inverse operations, and apply a variety of strategies for multiplication and division basic facts. They understand the meanings of multiplication and division of whole numbers through the use of various representations and situations (equal groups, arrays and area, multiplicative comparisons). They use properties of addition and multiplication to multiply whole numbers and apply increasingly sophisticated strategies based on these properties to solve multiplication and division problems involving basic facts (single-digit factors and related division facts). By comparing a variety of solution strategies, students relate multiplication and division as inverse operations.

Learning Goals and Performance (*Locally Assessed) Indicators:

1. Understand, explain, and apply meanings of multiplication and division.
 - a) Represent repeated addition as multiplication and repeated subtraction as division.
 - b) Show various representations and situations involving multiplication and division (equal-sized groups, arrays, area models, equal “jumps” on number lines, expressions and equations).
 - c) Describe multiplicative relationships in context (“on 1 chair, there are 4 legs, on 2 chairs 8 legs, etc;” “the number of legs is 4 times the number of chairs”).
 - d) *Use physical models and diagrams to demonstrate differences between partitive (sharing) and quotative (grouping) models of division.
 - e) *Compare solution strategies in order to relate multiplication and division as inverse operations.
2. Use and explain a variety of strategies to compute multiplication and division problems (basic facts with factors to 10).
 - a) *Compute multiplication basic facts (one-digit factors) and related division facts by using strategies based on properties (identity, commutative, associative, and distributive) and the inverse relationship of multiplication and division.
 - b) Use relational signs ($=$, \neq , $<$, $>$) and operational signs ($+$, $-$, \times , \div) to represent multiplication and division basic facts.
 - c) *Create problems for a variety of mathematical situations (addition, subtraction, one-digit factor multiplication and/or division [partitive and quotative]).
 - d) Solve problems for a variety of mathematical situations (addition, subtraction, one-digit factor multiplication and/or division [partitive and quotative]).

Grade 3, Core Content C: Meanings of Fractions and Fractional Relationships**25%**

Students develop an understanding of fractions and fraction equivalence by using area, length, and set models. They understand that the size of a fractional part is relative to the size of the whole, and they use fractions to represent numbers that are equal to, less than, or greater than 1. They solve problems that involve comparing and ordering fractions by using models, benchmark fractions, number lines or common numerators or denominators. They understand and use models to identify equivalent fractions.

Learning Goals and Performance (*Locally Assessed) Indicators:

1. Understand concepts of fractions equal to, less than, or greater than 1.
 - a) Recognize different representations of the same fraction (e.g., diagram, position on a number line, words, and numerical representation).
 - b) Represent fractions by dividing wholes into equal parts or defining regions, lines and sets of whole numbers.
 - c) Identify and apply the meaning of the denominator of a fraction as the number of equal parts of the unit whole and the numerator of a fraction as the number of equal parts being considered.
 - d) *Given a fraction, and its representation as a part of a whole, construct a whole.
2. Understand and use various methods to model and compare fractions and to identify equivalent fractions.
 - a) Compare and order common fractions in a variety of ways such as using physical models of fractions, relating fractions to benchmarks (e.g., closer to 0, $\frac{1}{2}$, or 1), number lines and comparing fractions with like numerators or denominators.
 - b) Use models, including the number line, to identify equivalent fractions.

Grade 3, Core Content D: Properties and Perimeter of Two-dimensional Shapes 20%

Students develop an understanding of properties and relationships in two-dimensional space and apply these ideas to measure real-world objects. They use benchmarks for classifying angles. They describe, analyze, compare, and classify two-dimensional shapes by their sides and angles and connect these attributes to the definitions of shapes. Students investigate, describe, and reason about composing and decomposing polygons to make other polygons. Through building, drawing, and analyzing two-dimensional shapes, students understand attributes and properties of two-dimensional space. They use these attributes and properties in solving problems, including applications involving perimeter.

Learning Goals and Performance (*Locally Assessed) Indicators:

1. Understand, explain, and use concepts of angle and angle measurement.
 - a) *Describe and interpret angles in various contexts, including static angles in geometric figures and dynamic angles involving motion (e.g., hands of a clock, the opening of a door).
 - b) *Develop references or benchmarks (45° , 90° , 180° , and 270°) for angle measures by using physical models such as pattern blocks and tangrams.
 - c) Classify angles as right, acute, or obtuse.
2. Understand that two-dimensional shapes can be created, described, compared, and classified on the basis of their attributes and properties.
 - a) Identify attributes classifying triangles (e.g., two equal sides for the isosceles triangle, right angle for the right triangle).
 - b) Identify attributes classifying quadrilaterals (e.g., parallel sides for the parallelogram, right angles or perpendicular sides for the rectangle).
 - c) Identify right, obtuse, and acute angles in geometric shapes.
 - d) *Predict the results of putting together and taking apart two-dimensional shapes, then test predictions with models.
 - e) *Create shapes satisfying particular specifications such as number of sides, relative length of sides, or relationships between sides (e.g., triangle with no congruent sides, pentagon with two parallel sides).
3. Understand, explain, and apply concepts of perimeter and perimeter measurement.
 - a) *Recognize that any point on a ruler can be used as a starting point for measurement, as long as adjustments are made for non-zero starting points.
 - b) *Use customary and metric units to estimate and measure the perimeter of real objects.
 - c) Use customary and metric units to determine the perimeter of polygons.
 - d) Create a variety of rectangles with a given perimeter.
 - e) Use perimeter concepts and knowledge of metric and customary measurement systems to solve problems involving rectangles, including the measure of an unknown side.

GRADE 4

A percentage of instructional time for each core content area is suggested. The sum of the percentages totals less than 100% so that school districts can modify the emphasis and/or add additional content in response to local needs.

Grade 4, Core Content A: Multiplication and Division: Basic Facts Fluency and Multi-digit Computation

30%

Students develop proficiency with multiplication and division basic facts, multi-digit multiplication, and division with single-digit divisors. They apply their understanding of models for multiplication and division, place value of numbers, and properties of operations (in particular, the distributive property) as they multiply and divide whole numbers. They select appropriate methods and apply them accurately to estimate products or calculate them mentally, depending on the context and numbers involved. They develop proficiency (accuracy, efficiency, flexibility, and appropriateness) with computational procedures, including a standard algorithm, for multiplying and dividing whole numbers, understand why these procedures work and use them to solve problems. In preparation for and as a result of working with larger numbers in multiplication and division, students also expand their understanding of place value.

Grade 4, Core Content B: Decimal Place Value and Connections to Fractions and Percents

30%

Students develop an understanding of decimals, including the connections among fractions, decimals, and percents. They understand decimal notation as an extension of the base-ten system of writing whole numbers that is useful for representing more numbers, including numbers between 0 and 1, between 1 and 2, and so on. Students relate their understanding of fractions to reading and writing decimals that are greater than or less than 1, identifying equivalent decimals, comparing and ordering decimals, and estimating decimal or fractional amounts in problem solving. They relate benchmark percents with decimals and connect equivalent fractions with decimals by comparing models to symbols and locating equivalent symbols on the number line.

Grade 4, Core Content C: Area Measurement and Transformations

15%

Students develop a basic understanding of area and transformations. They recognize area as an attribute of two-dimensional regions and learn that they can quantify area by finding the total number of same-sized units of area that cover the shape without gaps or overlaps. They understand that a plane-covering unit is needed to measure area and that a square which is 1 unit on a side is the standard unit for this measurement. They apply these ideas to measure plane figures and real-world objects (e.g., area of classroom floor). Students connect area measure to the area model that they have used to represent multiplication, and they use this connection to justify the formula for the area of a rectangle and triangle. Students transform two-dimensional shapes to check for congruence and symmetry (line and rotational).

Grade 4, Core Content D: Data Organization and Analysis

10%

Students develop an understanding of data analysis by formulating questions, collecting or using available data, analyzing tools of data representation, and using measures of center and spread to interpret data. They formulate and investigate questions that can be addressed with numerical data. They conduct simple experiments and use a variety of displays to represent and interpret data. Students attend to the distribution of data by describing its center and spread, and recognize limitations of the scope of inference beyond the experiment.

NOTE: The order of the Core Content, Learning Goals, and Performance Indicators on the pages that follow does not prescribe a sequence of instruction. Sequencing should be determined by individual school districts based on their local curricula and instructional programs.

Grade 4, Core Content A: Multiplication and Division Basic Facts and Multi-digit Computation 30%

Students develop proficiency with basic facts for multiplication and division and multi-digit multiplication and division with single-digit divisors. They apply their understanding of models for multiplication and division, place value of numbers, and properties of operations (in particular, the distributive property) as they multiply and divide whole numbers. They select appropriate methods and apply them accurately to estimate products or calculate them mentally, depending on the context and numbers involved. They develop proficiency (accuracy, efficiency, flexibility, and appropriateness) with computational procedures, including a standard algorithm, for multiplying and dividing whole numbers, understand why these procedures work and use them to solve problems. In preparation for and as a result of working with larger numbers in multiplication and division, students also expand their understanding of place value.

Learning Goals and Performance (*Locally Assessed) Indicators:

1. Understand and use efficient strategies for computing multiplication and division facts and use equations to represent the computations.
 - a) *Demonstrate proficiency computing multiplication and division facts (single-digit factors and related division facts).
 - b) Recognize and use relational ($=, \neq, <, >$) and operational ($+, -, \times, \div$) symbols to represent number sentences using multiplication and division.
 - c) Find the unknown quantity in a variety of number sentences (e.g., $20 \times 4 = \underline{\quad}$; $6 = 48 \div \underline{\quad}$; $4 \times 3 = \underline{\quad} \times 2$; $\underline{\quad} = 7 \times 8$) that involve addition, subtraction, multiplication and/or division.
2. Extend understanding of place value concepts and the operations of multiplication and division.
 - a) Determine the place value (hundred thousands through ones) and the value of each digit in a number (e.g., the 6 in 678,000 represents 6 hundred thousands, 60 ten thousands or 600 thousands) and translate among these representations.
 - b) Recognize problems that can be solved using multi-digit multiplication and use objects, diagrams (e.g., area models) and number sentences to represent and solve the problem.
 - c) *Describe the effects of multiplying whole numbers by 10, 100, or 1000.
 - d) * Explain what a remainder represents within the context of a problem involving division.
 - e) *Analyze a variety of strategies (including a standard algorithm) for multiplication and division in order to demonstrate their similarities and differences, and to draw conclusions about their efficiency, accuracy, and generalizability.
 - f) *Explain and justify multi-digit multiplication and division strategies on the basis of place value and properties of operations (identity, zero, commutative, associative, and distributive).
3. Understand, explain, and use efficient strategies to compute multiplication problems (with products to 10,000) and division problems (with up to three-digit dividend and one-digit divisor).
 - a) Multiply (multi-digit whole numbers) and divide (one-digit divisor and up to three-digit dividend) using efficient strategies, including a standard algorithm.
 - b) *Estimate products and/or calculate them mentally depending on the context and numbers involved; use estimates to judge the reasonableness of solutions.
 - c) *Create single- and multi-step problems for a variety of mathematical situations (multi-digit addition, multi-digit subtraction, multi-digit multiplication and/or division with one-digit divisor [partitive and quotative models]).
 - d) Solve single- and multi-step problems (multi-digit addition, multi-digit subtraction, multi-digit multiplication and/or division with one-digit divisor [partitive and quotative models]).

Grade 4, Core Content B: Decimal Place Value and Connections to Fractions and Percents 30%

Students develop an understanding of decimals, including the connections among fractions, decimals, and percents. They understand decimal notation as an extension of the base-ten system system of writing whole numbers that is useful for representing more numbers, including numbers between 0 and 1, between 1 and 2, and so on. Students relate their understanding of fractions to reading and writing decimals that are greater than or less than 1, identifying equivalent decimals, comparing and ordering decimals, and estimating decimal or fractional amounts in problem solving. They relate benchmark percents with decimals and connect equivalent fractions with decimals by comparing models to symbols and locating equivalent symbols on the number line.

Learning Goals and Performance (*Locally Assessed) Indicators:

1. Understand and apply the meaning of decimals and the relationships among the place values of their digits.
 - a) Represent hundredths and tenths using various models including base ten models, number line, words, standard and expanded forms.
 - b) Recognize and apply the concepts underlying place value (tenths, hundredths) by identifying the value of each digit in a decimal (e.g., the 7 in 0.78 represents 7 tenths or 70 hundredths) and translate among these representations.
 - c) Identify a tenth more or a tenth less, and a hundredth more or a hundredth less, than a given number.
2. Understand and explain relationships among commonly used fractions, decimals, and percents.
 - a) Model fractions (halves, fourths, fifths, and tenths) on a 10 x 10 grid and use this representation to convert fractions to decimals.
 - b) Recognize equivalent decimal forms for fractions with denominators of 10 and 100.
 - c) Recognize equivalent fraction and decimal forms for benchmark percents (0%, 25%, 50%, 75%, and 100%).
 - d) Identify equivalent fractions, decimals (less than one, equivalent to one, and greater than one) and percents, with and without models, including locations on a number line.
 - e) Use a variety of methods to compare and order fractions, decimals, and percents.

Grade 4, Core Content C: Area Measurement and Transformations**15%**

Students develop a basic understanding of area and transformations. They recognize area as an attribute of two-dimensional regions and learn that they can quantify area by finding the total number of same-sized units of area that cover the shape without gaps or overlaps. They understand that a plane-covering unit is needed to measure area and that a square which is 1 unit on a side is the standard unit for this measurement. They apply these ideas to measure plane figures and real-world objects (e.g., area of classroom floor). Students connect area measure to the area model that they have used to represent multiplication, and they use this connection to justify the formula for the area of a rectangle and triangle. Students transform two-dimensional shapes to check for congruence and symmetry (line and rotational).

Learning Goals and Performance (*Locally Assessed) Indicators:

1. Understand, explain, and apply the concepts of area and area measurement for rectangles and triangles and derive the area formula for rectangles and triangles.
 - a) Determine the area of polygons (rectangles, triangles, parallelograms) imposed on a grid by counting the number of units and half units.
 - b) *Derive and use the area formula for a rectangle and parallelogram ($A = b \cdot h$) and triangle ($A = \frac{1}{2}$ of area of related rectangle/parallelogram).
 - c) Identify the appropriate one- or two-dimensional unit (i.e. inches/feet, centimeters/meters, etc.) for a linear or area measurement.
 - d) Demonstrate that rectangles with the same area can have different perimeters (constant area, changing perimeter) and that rectangles with the same perimeter can have different areas (constant perimeter, changing area).
 - e) Using area and perimeter concepts, determine unknown side lengths (metric and customary) of rectangles.
2. Understand and explain the meanings and uses of transformations.
 - a) Use translations (slides), reflections (flips), and rotations (turns) to transform two-dimensional shapes.
 - b) Predict the result of a transformation.
 - c) Determine whether two shapes are congruent by using a combination of translations, reflections, and/or rotations with models.
 - d) Use transformations to determine whether a two-dimensional shape has line and/or rotational symmetry.

Grade 4, Core Content D: Data Organization and Analysis**10%**

Students develop an understanding of data analysis by formulating questions, collecting or using available data, analyzing tools of data representation, and using measures of center and spread to interpret data. They formulate and investigate questions that can be addressed with numerical data. They conduct simple experiments and use a variety of displays to represent and interpret data. Students attend to the distribution of data by describing its center and spread, and recognize limitations of the scope of inference beyond the experiment.

Learning Goals and Performance (*Locally Assessed) Indicators:

1. Understand how to formulate and answer questions that can be addressed by conducting simple experiments and collecting numerical data.
 - a) *Formulate questions that can be addressed with data.
 - b) Collect or use available numerical data in order to draw conclusions and answer questions.

2. Understand the tools for exploring distributions (including measures of center and spread), the appropriateness of data displays, and limitations of inference.
 - a) Determine the appropriate display (bar graph, line graph, line plot) to represent distributions of data.
 - b) Describe the distribution of data in terms of its center (mode and median) and spread (range).
 - c) Compare related data sets on the basis of measures of center (mode and median) and spread (range).
 - d) *Recognize limitations in the scope of inference beyond the experiment.

GRADE 5

A percentage of instructional time for each core content area is suggested. The sum of the percentages totals less than 100% so that school districts can modify the emphasis and/or add additional content in response to local needs.

Grade 5, Core Concept A: Division of Whole Numbers

30%

Students extend their understanding of and proficiency with whole number division and explain why strategies work on the basis of place value and properties of operations. They apply their understanding of place value, properties, models for division, and the relationship of division to multiplication as they find quotients involving multi-digit dividends. They select appropriate methods and apply them accurately to estimate quotients or calculate them mentally, depending on the context and numbers involved. They develop proficiency (accuracy, efficiency, flexibility, and appropriateness) with computational procedures, including a standard algorithm, for dividing whole numbers and they use these procedures to solve problems. They consider the context in which a problem is situated to select the most useful form of the quotient for the solution, and they interpret it appropriately.

Grade 5, Core Content B: Addition and Subtraction of Fractions and Decimals

30%

Students develop an understanding of and proficiency with addition and subtraction strategies for fractions (including mixed numbers) and decimals. They apply their understanding of fractions and fraction models to represent the addition and subtraction of fractions with like and unlike denominators, and their understanding of decimal models, place value, and properties to add and subtract decimals. They use appropriate methods to estimate sums and differences and consider estimates in judging the reasonableness of their solutions. They develop proficiency (accuracy, efficiency, flexibility, and appropriateness) with computational strategies (including standard algorithms) for adding and subtracting fractions and decimals. They explain why procedures work on the basis of place value and properties of operations. They add and subtract fractions and decimals to solve problems, including problems involving measurement.

Grade 5, Core Content C: Properties of Three-dimensional Shapes, Volume and Surface Area

15%

Students develop an understanding of the properties of three-dimensional shapes, including volume and surface area. They relate two-dimensional shapes to three-dimensional shapes and analyze properties of polyhedral solids, describing them by the number of edges, faces, or vertices as well as the types of faces. Students recognize volume as an attribute of three-dimensional space. They understand that they can quantify volume by finding the total number of same-sized units of volume that fill the space without gaps or overlaps. They understand that a space-filling unit is needed to measure volume and that a cube which is 1 unit on an edge is the standard unit for this measurement. They select appropriate units, strategies, and tools for solving problems that involve estimating or measuring volume. They decompose three-dimensional shapes and find surface areas and volumes of rectangular prisms.

Grade 5, Core Content D: Basic Probability Concepts

10%

Students develop an understanding of probability through the contexts of simple experiments and their outcomes. They begin to describe events as likely by using such words as *certain*, *equally likely*, and *impossible*. They learn that probability is a measurement of the likelihood of events and quantify the probability of an event as a number between 0 and 1. Moreover, they learn that sample space is the set of all possible outcomes and that the sum of the probabilities of all sample space outcomes is 1. They begin to quantify likelihood by conducting experiments that have only a few outcomes. Through these experiences, students encounter the idea that although they cannot determine an individual outcome, they can predict the frequency of various outcomes.

NOTE: The order of the Core Content, Learning Goals, and Performance Indicators on the pages that follow does not prescribe a sequence of instruction. Sequencing should be determined by individual school districts based on their local curricula and instructional programs.

Grade 5, Core Concept A: Division of Whole Numbers**30%**

Students extend their understanding of and proficiency with whole number division and explain why strategies work on the basis of place value and properties of operations. They apply their understanding of place value, properties, models for division, and the relationship of division to multiplication as they find quotients involving multi-digit dividends. They select appropriate methods and apply them accurately to estimate quotients or calculate them mentally, depending on the context and numbers involved. They develop proficiency (accuracy, efficiency, flexibility, and appropriateness) with computational procedures, including a standard algorithm, for dividing whole numbers and they use these procedures to solve problems. They consider the context in which a problem is situated to select the most useful form of the quotient for the solution, and they interpret it appropriately.

Learning Goals and Performance (*Locally Assessed) Indicators:

1. Understand multi-digit division (with divisors to 100 and dividends to 10,000).
 - a) Recognize problems that can be solved using multi-digit division and use objects, diagrams, and number sentences to represent and solve the problem.
 - b) Describe the effect of dividing by 10 or 100 on the quotient.
 - c) *Analyze a variety of strategies (including a standard algorithm) for division in order to demonstrate their similarities and differences, and to draw conclusions about their efficiency, accuracy, and generalizability.
 - d) *Explain and justify multi-digit division strategies on the basis of place value, relationships among operations, and properties of operations (identity, distributive).
2. Understand, explain, and use a variety of strategies to compute division problems proficiently (divisors to 100 and dividends to 10,000).
 - a) Divide whole numbers using standard algorithms and other efficient strategies.
 - b) Interpret the meaning of remainders expressed as whole numbers or fractions.
 - c) *Estimate products and quotients and/or calculate them mentally depending on the context and numbers involved; use estimates to judge the reasonableness of solutions.
 - d) Use relational ($=$, \neq , $<$, $>$) and operational signs ($+$, $-$, \times , \div) to represent a variety of equations and inequalities (e.g., $12 \div 4 = 3$; $121 \div 4 > 30$).
 - e) Find the unknown quantity in a variety of simple equations ($16 \times 4 = \underline{\quad}$; $24 = 48 \div \underline{\quad}$; $\underline{\quad} = 7 \times 21$; $4 \times 13 = \underline{\quad} \times 2$) that involve operations with whole numbers.
 - f) *Create single- and multi-step problems involving addition, subtraction, multiplication, and/or division of whole numbers.
 - g) Solve single- and multi-step problems involving addition, subtraction, multiplication, and/or division of whole numbers.

Grade 5, Core Content B: Addition and Subtraction of Fractions and Decimals**30%**

Students develop an understanding of and proficiency with addition and subtraction strategies for fractions (including mixed numbers) and decimals. They apply their understanding of fractions and fraction models to represent the addition and subtraction of fractions with like and unlike denominators, and their understanding of decimal models, place value, and properties to add and subtract decimals. They use appropriate methods to estimate sums and differences and consider estimates in judging the reasonableness of their solutions. They develop proficiency (accuracy, efficiency, flexibility, and appropriateness) with computational strategies (including standard algorithms) for adding and subtracting fractions and decimals. They explain why procedures work on the basis of place value and properties of operations. They add and subtract fractions and decimals to solve problems, including problems involving measurement.

Learning Goals and Performance (*Locally Assessed) Indicators:

1. Understand, explain, and apply strategies to add and subtract fractions proficiently (including mixed numbers with like and unlike denominators).
 - a) Recognize problems that can be solved by adding and/or subtracting fractions and use objects, diagrams, and number sentences to represent and solve the problems.
 - b) *Analyze a variety of strategies, including standard algorithms, for addition and subtraction of fractions in order to demonstrate their similarities and differences, and to draw conclusions about their efficiency, accuracy, and generalizability.
 - c) *Estimate fraction sums and differences and/or calculate them mentally, depending on the context and numbers involved; use estimates to judge the reasonableness of solutions.
 - d) Add and subtract fractions and mixed numbers with like and unlike denominators using standard algorithms and other efficient strategies.
 - e) *Create problems for a variety of mathematical situations (combining, missing addend, separating, comparing, and relating parts with wholes) involving fractions for which any one of the quantities is unknown.
 - f) Solve problems for a variety of mathematical situations involving fractions.
2. Understand, explain, and apply strategies to add and subtract decimals proficiently.
 - a) Recognize problems that can be solved by adding and/or subtracting decimals and use objects, diagrams, and number sentences to represent and solve the problems.
 - b) *Relate the addition and subtraction of decimals to the addition and subtraction of fractions.
 - c) *Analyze a variety of strategies, including standard algorithms, for addition and subtraction of decimals in order to demonstrate their similarities and differences, and to draw conclusions about their efficiency, accuracy, and generalizability.
 - d) Add and subtract decimal numbers using standard algorithms and other efficient strategies.
 - e) *Estimate decimal sums and differences and/or calculate them mentally, depending on the context and numbers involved; use estimates to judge the reasonableness of solutions.
 - f) *Create problems for a variety of mathematical situations involving decimal numbers.
 - g) Solve problems for a variety of mathematical situations involving decimal numbers.

Grade 5, Core Content C: Properties of Three-dimensional Shapes, Volume and Surface Area 15%

Students develop an understanding of the properties of three-dimensional shapes, including volume and surface area. They relate two-dimensional shapes to three-dimensional shapes and analyze properties of polyhedral solids, describing them by the number of edges, faces, or vertices as well as the types of faces. Students recognize volume as an attribute of three-dimensional space. They understand that they can quantify volume by finding the total number of same-sized units of volume that fill the space without gaps or overlaps. They understand that a space-filling unit is needed to measure volume and that a cube which is 1 unit on an edge is the standard unit for this measurement. They select appropriate units, strategies, and tools for solving problems that involve estimating or measuring volume. They decompose three-dimensional shapes and find surface areas and volumes of rectangular prisms.

Learning Goals and Performance (*Locally Assessed) Indicators:

1. Understand the properties of three-dimensional shapes.
 - a) Determine the number of edges, faces, and vertices of a given polyhedron.
 - b) Identify the types of faces of a given polyhedron.
 - c) *Compose larger polyhedra from smaller ones; decompose larger polyhedra into smaller ones.
 - d) Analyze and compare three-dimensional shapes on the basis of their edges, faces, and vertices.
 - e) Identify the two-dimensional shape (net) for a rectangular prism.
2. Understand and explain the concepts of surface area and volume as they relate to rectangular prisms.
 - a) Apply area concepts to solve problems involving surface areas (areas of nets) of rectangular prisms.
 - b) *Find the volume of rectangular prisms by using cubic units to fill them, with no gaps or overlaps, then counting the total number of units.
 - c) *Identify, organize, and use the underlying structure of cubes filling a rectangular prism (a series of layers) to find the volume of rectangular prisms.
 - d) *Solve problems that involve estimating or physically measuring the volume of rectangular prisms.
 - e) Determine the appropriate units for linear, area, and volume measurement situations.

Grade 5, Core Content D: Basic Probability Concepts**10%**

Students develop an understanding of probability through the contexts of simple experiments and their outcomes. They begin to describe events as likely by using such words as *certain*, *equally likely*, and *impossible*. They learn that probability is a measurement of the likelihood of events and quantify the probability of an event as a number between 0 and 1. Moreover, they learn that sample space is the set of all possible outcomes and that the sum of the probabilities of all sample space outcomes is 1. They begin to quantify likelihood by conducting experiments that have only a few outcomes. Through these experiences, students encounter the idea that although they cannot determine an individual outcome, they can predict the frequency of various outcomes.

Learning Goals and Performance (*Locally Assessed) Indicators:

1. Understand and explain that probability is a measurement of the likelihood of events and that the probability of an event is based on the set of all possible events called the sample space.
 - a) List all possible outcomes for simple experiments (e.g., predicting sums when rolling two number cubes).
 - b) Recognize whether an outcome of an experiment or simulation is *impossible*, *unlikely*, *possible*, *likely*, or *certain*, and whether two or more events are *equally likely*.
 - c) Represent the probability of an event, which ranges from 0 (*impossible*) to 1 (*certain*), with a fraction, decimal, or percent.
 - d) *Predict the likelihood of an outcome prior to an experiment and compare the predicted probability with the experimental results.

Core Content, Learning Goals, and Performance Indicators

Grades 6-8

DRAFT

GRADE 6

A percentage of instructional time for each core content area is suggested. The sum of the percentages totals less than 100% so that school districts can modify the emphasis and/or add additional content in response to local needs.

| | |
|--|------------|
| Grade 6, Core Content A: Multiplication and Division of Fractions and Decimals | 20% |
| Students use the meanings of fractions, multiplication and division, and the inverse relationship between multiplication and division to make sense of procedures for multiplying and dividing fractions and explain why they work. They use the relationship between decimals and fractions, as well as the relationship between finite decimals and whole numbers (i.e., a finite decimal multiplied by an appropriate power of 10 is a whole number), to understand and explain procedures for multiplying and dividing decimals. Students use common algorithms to multiply and divide fractions and decimals efficiently and accurately. They multiply and divide fractions and decimals to solve problems, including multi-step problems. | |
| Grade 6, Core Content B: Ratios, Rates, and Percents | 15% |
| Students use simple reasoning about multiplication and division to solve ratio and rate problems (e.g., “If 5 items cost \$3.75 and all items are the same price, then I can find the cost of 12 items by first dividing \$3.75 by 5 to find out how much one item costs and then multiplying the cost of a single item by 12”). By viewing equivalent ratios and rates as deriving from, and extending, pairs of rows (or columns) in the multiplication table, and by analyzing simple drawings that indicate the relative sizes of quantities, students extend whole number multiplication and division to ratios and rates. They expand the repertoire of problems that they can solve by using multiplication and division, and they build on their understanding of fractions to understand ratios. Students view percents as ratios and represent percents visually and numerically. They move flexibly between and among the percent, fraction, and decimal representations of numbers and use these representations in solving a variety of problems. | |
| Grade 6, Core Content C: Extending Properties and Measures of Two-dimensional Shapes | 20% |
| Students identify and classify triangles and quadrilaterals by their properties including angle measure, side lengths, parallelism, perpendicularity, and symmetry. They develop formulas for the areas of these shapes, reinforcing the connection between algebra and geometry. Students extend what they know about area and perimeter to more complex two-dimensional shapes including circles. Students use what they learn about triangles, quadrilaterals, and circles to solve a variety of problems. | |
| Grade 6, Core Content D: Mathematical Expressions and Equations | 15% |
| Students write mathematical expressions and equations that correspond to given situations, they evaluate expressions, and they use expressions and equations to solve problems. They understand that variables represent numbers whose exact values are not yet specified, and they use variables appropriately. Students understand that expressions in different forms can be equivalent, and they use properties to rewrite an expression to represent a quantity in a different way. Students know that the solutions to an equation are the values of the variables that make the equation true. They solve simple one-step equations by using number sense, properties of operations, and the idea of maintaining equality on both sides of an equation. | |
| Grade 6, Core Concept E: Sample Surveys and Data Distributions | 15% |
| Students formulate and answer questions by collecting and analyzing univariate numerical data, exploring methods of random selection, and recognizing the distinction among a population, a census, and a sample. They summarize and compare two or more distributions using a variety of displays and numerical summaries and describe differences between them with respect to center, spread, and shape, and recognize limitations in the scope of inference beyond the experiment. | |

NOTE: The order of the Core Content, Learning Goals, and Performance Indicators on the pages that follow does not prescribe a sequence of instruction. Sequencing should be determined by individual school districts based on their local curricula and instructional programs.

Grade 6, Core Content A: Multiplication and Division of Fractions and Decimals**10%**

Students use the meanings of fractions, multiplication and division, and the inverse relationship between multiplication and division to make sense of procedures for multiplying and dividing fractions and explain why they work. They use the relationship between decimals and fractions, as well as the relationship between finite decimals and whole numbers (i.e., a finite decimal multiplied by an appropriate power of 10 is a whole number), to understand and explain procedures for multiplying and dividing decimals. Students use common algorithms to multiply and divide fractions and decimals efficiently and accurately. They multiply and divide fractions and decimals to solve problems, including multi-step problems.

Learning Goals and Performance (*Locally Assessed) Indicators:

1. Develop proficiency in multiplying and dividing fractions and decimals.
 - a) *Estimate the results of computations with fractions and decimals and judge the reasonableness of the results.
 - b) Multiply and divide whole numbers and decimals by powers of ten (1000, 100, 10, 1, 0.1, 0.01, and 0.001).
 - c) Use the meanings of fractions and the inverse relationship between multiplication and division to make sense of procedures for multiplying and dividing with fractions.
 - d) Multiply and divide fractions and decimals using standard algorithms and other efficient strategies.
 - e) *Describe the effect of multiplying or dividing by a number between zero and one, by one, and by a number greater than one.
 - f) Solve single- and multi-step word problems involving multiplication and division of fractions and decimals.

Grade 6, Core Content B: Ratios, Rates, and Percents**15%**

Students use simple reasoning about multiplication and division to solve ratio and rate problems (e.g., “If 5 items cost \$3.75 and all items are the same price, then I can find the cost of 12 items by first dividing \$3.75 by 5 to find out how much one item costs and then multiplying the cost of a single item by 12”). By viewing equivalent ratios and rates as deriving from, and extending, pairs of rows (or columns) in the multiplication table, and by analyzing simple drawings that indicate the relative sizes of quantities, students extend whole number multiplication and division to ratios and rates. They expand the repertoire of problems that they can solve by using multiplication and division, and they build on their understanding of fractions to understand ratios. Students view percents as ratios and represent percents visually and numerically. They move flexibly between and among the percent, fraction, and decimal representations of numbers and use these representations in solving a variety of problems.

Learning Goals and Performance (*Locally Assessed) Indicators:

1. Understand and use ratios to represent quantitative relationships.
 - a) Identify and write ratios as comparisons of part-to-part and part-to-whole relationships using appropriate notation to describe problem situations.
 - b) *Justify why two different pairs of numbers may be used to represent the same ratio.
 - c) Solve single- and multi-step word problems involving ratios and rates.
 - d) Represent and model ratios associated with whole-number percents that are less than or equal to 100%.
2. Demonstrate flexibility with fractions, decimals, and percents.
 - a) Use efficient procedures to convert between and among fractional, decimal, and percent representations of number in mathematical and real-world situations.
 - b) Compare and order fractions, decimals, and percents and find their approximate locations on a number line.

Grade 6, Core Content C: Extending Properties and Measures of Two-dimensional Shapes 25%

Students identify and classify triangles and quadrilaterals by their properties including angle measure, side lengths, parallelism, perpendicularity, and symmetry. They develop formulas for the areas of these shapes, reinforcing the connection between algebra and geometry. Students extend what they know about area and perimeter to more complex two-dimensional shapes including circles. Students use what they learn about triangles, quadrilaterals, and circles to solve a variety of problems.

Learning Goals and Performance (*Locally Assessed) Indicators:

1. Represent, identify, and classify geometric shapes from written or verbal descriptions, measurements, and properties using sketches, figures represented on the coordinate plane, grids, or models.
 - a) Identify and use properties (including congruency, parallelism, perpendicularity, and symmetry) to classify quadrilaterals and triangles.
 - b) Draw or create two-dimensional shapes or models with specified measures and properties including the use of first-quadrant coordinates.
 - c) Measure angles using a protractor and other appropriate tools.
2. Apply formulas for perimeter and area of polygons and circles.
 - a) *Describe relationships between the areas of various polygons (e.g., parallelogram and rectangle or triangle and parallelogram with the same base and height) including how these relationships are expressed in the formulas for the figures.
 - b) Determine the area of triangles and quadrilaterals, with at least one pair of parallel sides, and composite figures including these shapes.
 - c) *Describe the relationship between the circumference and diameter of a circle (circumference = π x diameter) and use this relationship to develop general formulas such as $C = 2\pi r$ or $C = \pi d$ and $A = \pi r^2$.
 - d) Apply the formula for the circumference and area of circles to solve problems.

Grade 6, Core Content D: Polynomial Expressions and Equations**20%**

Students write polynomial expressions and equations that correspond to given situations, they evaluate and simplify expressions, and they use expressions and equations to solve problems. They understand that variables represent numbers whose exact values are not yet specified, and they use variables appropriately. Students understand that expressions in different forms can be equivalent, and they use properties to rewrite an expression to represent a quantity in a different way. Students know that the solutions to an equation are the values of the variables that make the equation true. They solve simple one-step equations by using number sense, properties of operations, and the idea of maintaining equality on both sides of an equation.

Learning Goals and Performance (*Locally Assessed) Indicators:

1. Write polynomial expressions and equations that correspond to a given situation; evaluate and simplify expressions.
 - a) Write mathematical expressions and equations with variables to represent a given situation.
 - b) Evaluate mathematical expressions using properties (associative, commutative, distributive) and order of operations.
 - c) Evaluate mathematical expressions when the value for each variable is given.
 - d) *Solve simple equations generated from representing situations mathematically using informal strategies (e.g., guess and check, working backwards) and verify solutions.

Grade 6, Core Concept E: Sample Surveys and Data Distributions**15%**

Students formulate and answer questions by collecting and analyzing univariate numerical data, exploring methods of random selection, and recognizing the distinction among a population, a census, and a sample. They summarize and compare two or more distributions using a variety of displays and numerical summaries and describe differences between them with respect to center, spread, and shape, and recognize limitations in the scope of inference beyond the experiment.

Learning Goals and Performance (*Locally Assessed) Indicators:

1. Formulate questions, identify the numerical attributes on which to collect data, decide how to measure the attribute, determine and implement a data collection process.
 - a) *Design, conduct, and evaluate sample surveys.
 - b) *Distinguish between a *population* and *sample* and recognize potential sources of bias associated with sample selection.

2. Compare two or more data distributions using displays and numerical summaries and describe differences between them with respect to center, spread, and shape. Recognize limitations in the scope of inference beyond the experiment.
 - a) Summarize and compare distributions using numerical summaries and data displays, including relative frequency tables, box plots, circle graphs (pie charts), back-to-back stem-and-leaf plots, double bar graphs, and histograms.
 - b) Find measures of center (mean, median, and mode) and justify which is most appropriate for summarizing a given data set given the context.
 - c) *Describe how mean, median, mode, or range relates to the shape of the distribution.
 - d) *Distinguish between interpretations of the mean as the “fair share” value for data and as the “balancing point” of the corresponding data distribution.

GRADE 7

A percentage of instructional time for each core content area is suggested. The sum of the percentages totals less than 100% so that school districts can modify the emphasis and/or add additional content in response to local needs.

Grade 7, Core Content A: Rational Numbers and Linear Equations

30%

Students extend understandings of addition, subtraction, multiplication, and division, together with their properties, to all rational numbers, including negative integers. By applying properties of arithmetic and considering negative numbers in everyday contexts students explain why the rules for adding, subtracting, multiplying, and dividing with negative numbers make sense. Students use the arithmetic of rational numbers as they formulate and solve linear equations in one variable and use these equations to solve problems. Students make strategic choices of procedures to solve linear equations in one variable and implement them efficiently, understanding that when they use the properties of equality to express an equation in a new way, solutions that they obtain for the new equation also solve the original equation.

Grade 7, Core Content B: Proportionality and Similarity

25%

Students extend their work with ratios to develop an understanding of proportionality that they apply to solve single- and multi-step problems in numerous contexts. They use ratio and proportionality to solve a wide variety of percent problems, including problems involving discounts, interest, taxes, tips, and percent of increase or decrease. They also solve problems about similar objects (including figures) by using scale factors that relate corresponding lengths of the objects or by using the fact that relationships of lengths within an object are preserved in similar objects. Students graph proportional relationships and identify the unit rate as the slope of the related line. They distinguish proportional relationships ($y/x = K$ or $y = kx$) from other relationships, including inverse proportionality ($xy = k$ or $y = k/x$).

Grade 7, Core Content C: Surface Area and Volume

20%

By decomposing two- and three-dimensional shapes into smaller, component shapes, students find surface areas and develop and justify formulas for the surface areas and volumes of prisms and cylinders. As students decompose prisms and cylinders by slicing them, they develop and understand formulas for their volumes (Volume = Area of base \times Height). They apply these formulas in problem solving to determine volumes of prisms and cylinders. Students investigate how surface area and volume are affected when one or more dimensions of a figure are changed. They select appropriate two- and three-dimensional shapes to model real-world situations, create two-dimensional drawings of three-dimensional figures, and solve a variety of problems (including multi-step problems) involving surface areas and volumes of prisms and cylinders.

Grade 7, Core Concept D: Experimental and Theoretical Probability

10%

Students conduct one-stage experiments, use experimental data to estimate probabilities, and, where possible, compare experimental and theoretical probabilities and examine experimental probability in the long run. Students recognize that repetitions of an experiment may result in different outcomes and describe the variation in outcomes produced in an experiment. Students recognize that small samples are often not representative of the population from which they are drawn, and with the collection of more data the experimental probability of a particular outcome approaches the theoretical probability.

NOTE: The order of the Core Content, Learning Goals, and Performance Indicators on the pages that follow does not prescribe a sequence of instruction. Sequencing should be determined by individual school districts based on their local curricula and instructional programs.

Grade 7, Core Content A: Rational Numbers and Linear Equations**30%**

Students extend understandings of addition, subtraction, multiplication, and division, together with their properties, to all rational numbers, including negative rational numbers. By applying properties of arithmetic and considering negative numbers in everyday contexts students explain why the rules for adding, subtracting, multiplying, and dividing with negative numbers make sense. Students use the arithmetic of rational numbers as they formulate and solve linear equations in one variable and use these equations to solve problems. Students make strategic choices of procedures to solve linear equations in one variable and implement them efficiently, understanding that when they use the properties of equality to express an equation in a new way, solutions that they obtain for the new equation also solve the original equation.

Learning Goals and Performance (*Locally Assessed) Indicators:

1. Represent, compare, order, and compute with all rational numbers. Solve problems involving rational numbers.
 - a) *Explain everyday contexts where integers are used to quantify situations (e.g., owing money, measuring elevations above and below sea level).
 - b) Compare and order rational numbers including positive and negative integers. Find their approximate location on a number line and use symbols ($<$, $>$, $=$) to represent comparisons.
 - c) Use absolute value notation and know that the absolute value of an integer is its distance from zero on a number line.
 - d) *Show how operations on integers can be modeled and use the models to develop and explain efficient procedures for computing with integers.
 - e) Add, subtract, multiply, and divide all rational numbers.

2. Represent situations symbolically and solve problems that involve linear relationships.
 - a) Represent linear relationships with equations using both explicit and recursive (Next, Now) notation.
 - b) Solve one- and two-step linear equations (one unknown) with integer coefficients.
 - c) Write an equation that corresponds to a given problem situation, and describe a problem situation that corresponds to a given equation.
 - d) Solve single- and multi-step word problems involving rational numbers and verify the solutions.

Grade 7, Core Content B: Proportionality and Similarity**25%**

Students extend their work with ratios to develop an understanding of proportionality that they apply to solve single- and multi-step problems in numerous contexts. They use ratio and proportionality to solve a wide variety of percent problems, including problems involving discounts, interest, taxes, tips, and percent of increase or decrease. They also solve problems about similar objects (including figures) by using scale factors that relate corresponding lengths of the objects or by using the fact that relationships of lengths within an object are preserved in similar objects. Students graph proportional relationships and identify the unit rate as the slope of the related line. They distinguish proportional relationships ($y/x = K$ or $y = kx$) from other relationships, including inverse proportionality ($xy = k$ or $y = k/x$).

Learning Goals and Performance (*Locally Assessed) Indicators:

1. Develop computational fluency in working with ratios, percents, and proportional situations and apply this fluency to estimate the solution to and solve a variety of problems.
 - a) Use proportionality to model and solve problems, including percent applications and measurement conversions.
 - b) *Estimate solutions to percent problems.
 - c) Use proportionality to interpret circle graphs.
2. Identify, describe, and apply similarity relationships to find measures of corresponding parts in similar figures and apply scales/scale factors to measurements in drawings and maps.
 - a) Given similar two-dimensional shapes, identify the scale factor and describe the relationships between the scale factor and measurements of corresponding parts (angles, side lengths, perimeters, areas).
 - b) Determine if two shapes are similar and justify the conclusion by examining corresponding side lengths, angles, perimeters, and area.
 - c) *Interpret and solve scaling problems involving various mathematical contexts (e.g. indirect measurement, scale models).
3. Connect the constant rate of change in a proportional relationship to the concept of slope of a line.
 - a) *Represent proportional relationships using graphs, tables, verbal descriptions, and equations, and make connections among the representations.
 - b) *Determine the slope/rate of change of a line corresponding to the graph of a proportional relationship, recognize that slope is the same between any two points on the line, and that similar triangles may be used to demonstrate constant slope.
 - c) *Determine the unit rate in a proportional relationship and relate it to the slope of the associated line.

Grade 7, Core Content C: Surface Area and Volume**20%**

By decomposing two- and three-dimensional shapes into smaller, component shapes, students find surface areas and develop and justify formulas for the surface areas and volumes of prisms and cylinders. As students decompose prisms and cylinders, they develop and understand formulas for their volumes (Volume = Area of base \times Height) and surface areas. They apply these formulas in problem solving to determine volumes of prisms and cylinders. Students investigate how surface area and volume are affected when one or more dimensions of a figure are changed. They select appropriate two- and three-dimensional shapes to model real-world situations, create two-dimensional drawings of three-dimensional figures, and solve a variety of problems (including multi-step problems) involving surface areas and volumes of prisms and cylinders.

Learning Goals and Performance (*Locally Assessed) Indicators:

1. Describe the components of two- and three-dimensional shapes.
 - a) Name and identify attributes of two- and three-dimensional shapes including right triangles (legs and hypotenuse), circle or sphere dimensions (radius, diameter), and surfaces of rectangular prisms (base, faces, vertices, edges).
 - b) Describe the surfaces of common three-dimensional shapes (e.g., base of a cylinder is a circle; face of a cylinder is a rectangle) including cylinders, cones, rectangular prisms, pyramids.
 - c) Describe the cross section (parallel to the base) of common three-dimensional shapes (e.g., cylinder, cone, rectangular prism).
 - d) Identify the measurable features of three-dimensional shapes (size of base, height, surface area, volume) and the types of units used to measure each feature.
2. Develop formulas for measuring surface area and volume of common three-dimensional shapes.
 - a) *Describe the relationships between the measurements of three-dimensional figures and the measures of related two-dimensional shapes (e.g., the volume of a rectangular prism can be found by multiplying the base area by the height.)
 - b) *Recognize and draw two-dimensional representations (isometric and perspective drawings) of three-dimensional shapes.
 - c) Determine the surface area and volume of right prisms and right cylinders.
 - d) *Describe the relationship between the volume of right prisms and pyramids, and cylinders and cones.
 - e) Describe how surface area and volume are affected when a shape's linear dimensions are changed by a whole unit or unit fraction scale factor.
 - f) Describe how surface area and volume are affected when a shape's linear dimensions are changed by a scale factor (whole number or unit fraction).
3. Understand the concepts of "square root" and "cube root" and use the terminology and symbols associated with these concepts. Make estimates of square and cube roots and know common whole number square and cube roots.
 - a) Explain the meaning of "square root" and "cube root" and represent each using appropriate symbols.
 - b) Estimate square root of numbers less than 225 and cube root of numbers less than 1000 to the nearest whole number.
 - c) *Recall the square roots of perfect squares between 1 and 100 and the cube roots of perfect cubes from 1 through 1000.
 - d) *Solve area and volume problems using square and cube roots.

Grade 7, Core Concept D: Experimental and Theoretical Probability**10%**

Students conduct one-stage experiments, use experimental data to estimate probabilities, and, where possible, compare experimental and theoretical probabilities and examine experimental probability in the long run. Students recognize that repetitions of an experiment may result in different outcomes and describe the variation in outcomes produced in an experiment. Students recognize that small samples are often not representative of the population from which they are drawn, and with the collection of more data the experimental probability of a particular outcome approaches the theoretical probability.

Learning Goals and Performance (*Locally Assessed) Indicators:

1. Conduct one-stage experiments to estimate the likelihood of a simple event, compare the experimental probability with an easily identifiable theoretical probability, describe and compare the likelihood of events.
 - a) Determine the sample space for a given one-stage experiment using lists, tables, and tree diagrams to represent all possible outcomes.
 - b) Within a given context, determine the theoretical probability of an event and its *complement* given a sample space.
 - c) *Recognize that with the collection of more data, the experimental probability of a particular outcome approaches the theoretical probability and that although probability cannot determine an individual outcome, it can be used to predict the frequency of an outcome.
 - d) Use experimental data to estimate the probability of an event when the theoretical probability is unknown.

GRADE 8

A percentage of instructional time for each core content area is suggested. The sum of the percentages totals less than 100% so that school districts can modify the emphasis and/or add additional content in response to local needs.

Grade 8, Core Content A: Real Numbers, Polynomial Expressions and Operations on Polynomials Expressions

15%

Students see the number system extended to the real numbers represented by the number line. They determine whether approximations or exact values of real numbers are appropriate, depending on the context, and justify the selection. They work with integer exponents, scientific notation, and radicals, and use variables and expressions to solve problems from purely mathematical as well as applied contexts. They demonstrate ability to write and manipulate a variety of algebraic expressions as they apply algebraic procedures to solve problems.

Grade 8, Core Content B: Linear Functions and Equations

25%

Students build on their work with proportions and linear equations to work with a broader set of linear relationships, including functions. Students understand that the slope of a line is a constant rate of change. They move flexibly between and among verbal, tabular, graphical, and algebraic representations of functions (recognizing that tabular and graphical representations are usually only partial representations). Students describe how the slope and y-intercept of a function appear in the tabular, graphical, and algebraic representations of the function.

Grade 8, Core Content C: Transformations

15%

Students investigate the effects of transformations on geometric figures. They represent translations, reflections, rotations, and simple compositions of these transformations using sketches and coordinates, and explain the effects of these transformations. Students recognize and identify corresponding parts of the pre-image and image and recognize that these figures are similar. Students understand that symmetry results from transformations and identify lines of symmetry and angles of rotation.

Grade 8, Core Content D: Distance and Angle Measurement

20%

Students use facts about distance and angles to describe and analyze figures and situations in two- and three-dimensional space and to solve problems, including those with multiple steps. They explore the relationships between pairs of angles formed when parallel lines are cut by a transversal and use these relationships to find unknown measures of angles. Students connect geometry and measurement as they use relationships involving the sides and angles of triangles to finding unknown measures. Students explain why the Pythagorean theorem is valid by using a variety of methods—for example, by decomposing a square in two different ways. They apply the Pythagorean theorem to find distances between points in the Cartesian coordinate plane, to measure lengths, and to analyze polygons and polyhedra.

Grade 8, Core Content E: Bivariate Data

15%

Students formulate questions involving two attributes, design experiments, collect, organize, and summarize bivariate data using numerical and graphical displays. Students quantify the strength of association between two variables, develop simple models for association, and understand basic interpretations of measures of association.

NOTE: The order of the Core Content, Learning Goals, and Performance Indicators on the pages that follow does not prescribe a sequence of instruction. Sequencing should be determined by individual school districts based on their local curricula and instructional programs.

Grade 8, Core Content A: Real Numbers, Polynomial Expressions and Operations on Polynomials Expressions

15%

Students see the number system extended to the real numbers represented by the number line. They determine whether approximations or exact values of real numbers are appropriate, depending on the context, and justify the selection. They work with integer exponents, scientific notation, and radicals, and use variables and expressions to solve problems from purely mathematical as well as applied contexts. They demonstrate ability to write and manipulate a variety of algebraic expressions as they apply algebraic procedures to solve problems.

Learning Goals and Performance (*Locally Assessed) Indicators:

1. Develop an understanding of the real number system, including the notation associated with negative numbers, irrational numbers, and large and small numbers.
 - a) Compare and order real numbers, find their approximate locations on the number line.
 - b) Identify a real number as rational or irrational based on the definition of rational numbers (numbers that can be expressed in the form a/b where a and b are integers and b is not equal to 0).
 - c) Convert rational numbers to terminating or repeating decimal form and use appropriate notation to indicate the repeating digit sequence.
 - d) Extend/adapt algorithms for computing with positive rational numbers to negative rational numbers.
 - e) Represent and solve problems involving rational numbers and judge the reasonableness of solutions.
2. Use exponents in expressions containing both numbers and variables.
 - a) Use exponents to represent repeated multiplication and calculate the value of expressions represented with exponential notation.
 - b) Represent numbers in scientific notation, and translate numbers written in scientific notation into standard form.
 - c) Evaluate numerical expressions involving integer exponents.
 - d) Simplify expressions involving integer exponents using the laws of exponents.
3. Use variables to represent unknown values, evaluate simple expressions and operate on polynomials.
 - a) Determine all possible values of variables that satisfy prescribed conditions and evaluate algebraic expressions that involve variables.
 - b) Evaluate expressions and solve equations involving variables when values for the variables are given.
 - c) Add and subtract polynomials; multiply a monomial by a polynomial; divide a polynomial by a monomial.

Grade 8, Core Content B: Linear Functions and Equations**25%**

Students build on their work with proportions and linear equations to work with a broader set of linear relationships, including functions. Students understand that the slope of a line is a constant rate of change. They move flexibly between and among verbal, tabular, graphical, and algebraic representations of functions (recognizing that tabular and graphical representations are usually only partial representations). Students describe how the slope and y-intercept of a function appear in the tabular, graphical, and algebraic representations of the function.

Learning Goals and Performance (*Locally Assessed) Indicators:

1. Represent situations and solve problems that involve linear equations and linear functions.
 - a) Solve linear equations with rational coefficients and one unknown.
 - b) Solve single- and multi-step word problems involving linear equations and verify the solutions.
 - c) Solve one- and two-step linear inequalities with rational coefficients and one unknown.
 - d) Represent and describe linear functions using verbal descriptions, tables, graphs, or symbolic expressions, and make connections among these representations.
 - e) Determine the slope and y-intercept of a linear function described by a symbolic expression, table, verbal description or graph.
 - f) Interpret the slope and y-intercept of a graph of a linear function.
 - g) *Describe the domain and range of a function.

Grade 8, Core Content C: Transformations**15%**

Students investigate the effects of transformations on geometric figures. They represent translations, reflections, rotations, and simple compositions of these transformations using sketches and coordinates, and explain the effects of these transformations. Students recognize and identify corresponding parts of the pre-image and image and recognize that these figures are similar. Students understand that symmetry results from transformations and identify lines of symmetry and angles of rotation.

Learning Goals and Performance (*Locally Assessed) Indicators:

1. Apply knowledge of transformations in relating objects and in their effects on figures.
 - a) Locate the images of figures produced by transformations including translations, reflections about a vertical or horizontal line, rotations about the origin, and simple composition of these transformations.
 - b) *Describe the effects of transformations including translations, reflections about a vertical or horizontal line, rotations about the origin, and simple composition of these transformations.
 - c) Determine the coordinates of the image of a figure produced after a translation, a reflection about a vertical or horizontal line, or a rotation of a multiple of 90° about the origin of the coordinate plane.
 - d) Identify the corresponding parts (segments, angles, vertices) of the pre-image and image of a figure and identify the transformation that has occurred.
 - e) Draw lines of symmetry and identify the angle of rotation in designs with rotational symmetry.

Grade 8, Core Content D: Distance and Angle Measurement**20%**

Students use facts about distance and angles to describe and analyze figures and situations in two- and three-dimensional space and to solve problems, including those with multiple steps. They explore the relationships between pairs of angles formed when parallel lines are cut by a transversal and use these relationships to find unknown measures of angles. Students connect geometry and measurement as they use relationships involving the sides and angles of triangles to finding unknown measures. Students explain why the Pythagorean theorem is valid by using a variety of methods—for example, by decomposing a square in two different ways. They apply the Pythagorean theorem to find distances between points in the Cartesian coordinate plane, to measure lengths, and to analyze polygons and polyhedra.

Learning Goals and Performance (*Locally Assessed) Indicators:

1. Develop and apply the Pythagorean Theorem to solve for the lengths of sides in rights triangles and related measurement problems.
 - a) *State the Pythagorean Theorem and justify it using a variety of methods.
 - b) Apply the Pythagorean Theorem and its converse to solve problems.
 - c) Apply the Pythagorean Theorem to determine the distance between two points on the coordinate plane.
2. Solve problems involving angle measure.
 - a) Identify pairs of angles as complementary, supplementary, adjacent, or vertical and find missing angle measures by using these relationships.
 - b) Find the measure of angles formed when parallel lines are cut by a transversal and at least one angle measure is given.
 - c) Identify, state, and apply the Angle-Sum properties for triangles and other polygons.

Grade 8, Core Content E: Bivariate Data**15%**

Students formulate questions involving two attributes, design experiments, collect, organize, and summarize bivariate data using numerical and graphical displays. Students quantify the strength of association between two variables, develop simple models for association, and understand basic interpretations of measures of association.

Learning Goals and Performance (*Locally Assessed) Indicators:

1. Formulate questions involving two attributes, design experiments, collect, organize, and summarize bivariate data using numerical and graphical displays to represent the data.
 - a) *Design experiments and collect bivariate (two-variable) data to answer a question, classifying each attribute as a categorical or numerical variable.
 - b) Identify, describe, and construct appropriate displays (*two-way tables*, *parallel box plots* or *back-to-back stem-and-leaf plots*, and *scatterplots*) for bivariate data.
 - c) Summarize data using statistics including five-number summaries and *Inter-Quartile Range (IQR)*.
2. Quantify the strength of association between two variables using a variety of tools, develop simple models for association between two numerical variables, and understand basic interpretations of measures of association.
 - a) Quantify the strength of association between two variables and develop simple models using contingency tables (for categorical data) and the Quadrant Count Ratio (for numerical data) and simple models for examining the association between two numerical values such as simple lines (e.g., median-median line).
 - b) *Describe the relationship between the two variables, the effects of outliers on the observed relationship, and distinguish between an “association” and “cause and effect.”

Core Content, Learning Goals, and Performance Indicators
High School

DRAFT

ALGEBRA I

A percentage of instructional time for each core content area is suggested. The sum of the percentages totals less than 100% so that school districts can modify the emphasis and/or add additional content in response to local needs.

| | |
|---|------------|
| Algebra I, Core Content A: Expressions, Linear Equations, and Inequalities | 30% |
| Students further develop ability to represent linear functions using symbolic notation and to solve linear equations and inequalities. They generate equivalent algebraic expressions and equations involving polynomials and rational polynomial expressions. They maintain computational fluency with rational numbers by applying algorithms to operate on and simplify rational expressions. They construct, represent, solve, and interpret solutions of linear equations, absolute value equations, linear inequalities, and systems of linear equations. They determine the reasonableness of solution(s). | |

| | |
|---|------------|
| Algebra I, Core Content B: Functions | 20% |
| Students understand the concept of function and identify important features of functions and other relations. They represent functions using algebraic, graphical, tabular, and verbal representations (recognizing that tabular and graphical representations are usually only partial representations of the function). They move flexibly between and among representations. They apply the terminology and symbols associated with expressions, functions, and linear equations, including domain, range, slope, intercepts, independent and dependent variables. | |

| | |
|--|------------|
| Algebra I, Core Content C: Exponential, Quadratic and Other Nonlinear Equations | 25% |
| Students recognize quadratic, exponential and other common functions in real-world and mathematical situations. They represent these functions with tables, verbal descriptions, symbols, and graphs. They solve problems involving these functions and explain results in the original context. | |

| | |
|--|------------|
| Algebra I, Core Content D: Trend Lines and Correlation | 10% |
| Students develop models for trends in bivariate data using both median-fit lines and least-squares regression lines. They use the correlation coefficient to measure linear association in scatterplots. Students examine the effects of outliers on the models for trend and on correlation. Students extend their understanding of data analysis through modeling and interpreting problems. | |

NOTE: The order of the Core Content, Learning Goals, and Performance Indicators on the pages that follow does not prescribe a sequence of instruction. Sequencing should be determined by individual school districts based on their local curricula and instructional programs.

Algebra I, Core Content A: Expressions, Linear Equations, and Inequalities**30%**

Students further develop ability to represent linear functions using symbolic notation and to solve linear equations and inequalities. They generate equivalent algebraic expressions and equations involving polynomials and rational polynomial expressions. They maintain computational fluency with rational numbers by applying algorithms to operate on and simplify rational expressions. They construct, represent, and solve linear equations, absolute value equations, linear inequalities, and systems of linear equations. They determine the reasonableness of solution(s).

Learning Goals and Performance (*Locally Assessed) Indicators:

1. Simplify expressions to generate equivalent forms to interpret and solve problems.
 - a) Factor common monomial factors from polynomials and factor quadratic polynomials.
 - b) Use the laws of exponents to simplify and evaluate numeric and algebraic expressions.
 - c) Represent square and cube roots using rational exponents.
 - d) Simplify square roots.
 - e) Simplify algebraic expressions, including those involving simple rational expressions [e.g., $2/x + 1/(1+x)$], and justify that two algebraic expressions, equations, or inequalities are equivalent.
 - f) Simplify absolute value expressions.
 - g) Add, subtract, multiply polynomials; divide a polynomial by a polynomial of equal or lower degree.
2. Solve and represent solutions of linear equations and inequalities. Understand the relationship between slopes of lines in the plane that are parallel, perpendicular, or neither.
 - a) Graph the line that represents solutions of a linear equation on the x-y coordinate plane.
 - b) Write equations and graph lines given particular information (e.g., two points on a line or slope and one point on the line).
 - c) Recognize and use different forms of linear equations including slope-intercept, point-slope, intercept, and general (standard) forms.
 - d) Identify the slope and x- and y-intercepts of a line given the equation of the line.
 - e) Recognize the relationship between the slopes of parallel lines and between perpendicular lines and use these relationships to identify and/or write equations for lines parallel or perpendicular to a given line.
 - f) Construct a linear equation or linear inequality to model a mathematical or real-world situation.
 - g) Solve linear equations and inequalities (one variable) using symbolic methods, graphs, tables, and technology.
 - h) Solve linear absolute value equations.
3. Construct, solve, and interpret solutions of systems of linear equations (limited to two equations with two unknowns) that model mathematical or real-world situations.
 - a) *Construct a system of linear equations modeling a mathematical or real-world situation.
 - b) *Graph a system of linear equations and indicate possible solutions (intersection points).
 - c) *Solve a system of two linear equations and describe the nature of the solutions (no solution, one solution, or infinitely many solutions).

Algebra I, Core Content B: Functions**20%**

Students understand the concept of function and identify important features of functions and other relations. They represent functions using algebraic, graphical, tabular, and verbal representations (recognizing that tabular and graphical representations are usually only partial representations of the function). They move flexibly between and among representations. They apply the terminology and symbols associated with expressions, functions, and linear equations, including domain, range, slope, intercepts, independent, and dependent variables.

Learning Goals and Performance (*Locally Assessed) Indicators:

1. Understand the definition of a function and identify important features of functions and other relations using symbolic and graphical methods as appropriate.
 - a) Distinguish between functions and other relations defined symbolically, graphically, or in tabular form.
 - b) Represent a function with a symbolic expression, as a graph, in a table, and using words, and make connections among these representations.
 - c) Find the domain and range of a function defined symbolically, graphically, or in a real-world context.
 - d) Compare linear and nonlinear functions by examining rates of change from verbal descriptions, tables of values, graphical representations, and symbolic forms.
2. Describe and represent functions using appropriate notation and terminology, and evaluate functions at given points.
 - a) Represent linear patterns or arithmetic sequences using verbal rules and symbolic expressions such as kx (proportional relationships) and $ax + b$ (more-general linear relationships).
 - b) Analyze a mathematical or real-world situation, determine whether a linear function can describe the situation, and if so, write the linear function.
 - c) Describe examples of situations represented by piecewise-linear functions.
 - d) Use functional notation and evaluate a function at a given point in its domain (e.g., If $f(x) = 2x - 3$, find $f(-2)$).

Algebra I, Core Content C: Exponential, Quadratic, and Other Nonlinear Equations**25%**

Students recognize quadratic, exponential and other common functions in real-world and mathematical situations. They represent these functions with tables, verbal descriptions, symbols, and graphs. They learn various strategies and techniques for solving quadratic equations including factoring, completing the square, and quadratic formula. They solve problems involving these functions and explain results in the original context.

Learning Goals and Performance (*Locally Assessed) Indicators:

1. Identify certain nonlinear relationships and classify them as exponential relationships, quadratic relationships, or relationships of the form $y = k/x$, based on rates of change in tables, symbolic forms, or graphical representations.
 - a) Identify nonlinear (exponential, quadratic, and equations of the form $y = k/x$) relationships in graphical or tabular displays through an examination of successive differences, ratios, symbolic forms, or graphical properties.
 - b) *Solve for terms in a geometric (exponential) sequence using given verbal rules or symbolic expressions (explicit and recursive).
 - c) Recognize exponential functions from their verbal descriptions and tabular, graphical or symbolic representations, and move flexibly between and among these representations.
 - d) Describe the effects of changes in the coefficient, base, and exponent on the growth described by an exponential function.
2. Represent and solve quadratic equations.
 - a) Represent a quadratic equation with a symbolic expression, as a graph, in a table, and with a description, and make connections among the representations.
 - b) Sketch the graph of a quadratic equation, describe the effects that changes in the parameters have on the graph, and interpret the x -intercepts as solutions to a quadratic equation.
 - c) Solve quadratic equations by factoring, completing the square, or using the quadratic formula.
3. Represent simple exponential functions using tables, symbolic forms, or graphical representations.
 - a) Sketch the graph for an exponential function of the form $y = ab^x$ where x is an integer.
 - b) Describe the effects that changes in the parameters a and b have on the graph of an exponential function of the form $y = ab^n$ where n is an integer.
 - c) Find and approximate solutions to exponential equations.
 - d) Distinguish among general forms for exponential equations ($y = b^x$, $y = a \cdot b^x$) and quadratic equations ($y = x^2$, $y = -x^2$, $y = ax^2$, $y = x^2 + c$, $y = ax^2 + c$) and describe how the values of a , b , and c affect their graphical representations.

Algebra I, Core Content D: Trend Lines and Correlation**10%**

Students develop models for trends in bivariate data using both median-fit lines and least-squares regression lines. They use the correlation coefficient to measure linear association in scatterplots. Students examine the effects of outliers on the models for trend and on correlation. Students extend their understanding of data analysis through modeling and interpreting problems related to mathematical or real-world situations.

Learning Goals and Performance (*Locally Assessed) Indicators:

1. Model association of bivariate numerical data using tables and scatter plots and apply the correlation coefficient to measure and describe linear association. Apply both median-fit lines and least-squares regression lines to describe trends in bivariate data, when appropriate.
 - a) Compute the median-median line to model a relationship shown in a scatter plot, and interpret the slope and intercept in terms of the original context.
 - b) *Generate the least-squares regression line, using technology, to model a relationship shown in a scatter plot.
 - c) Interpret the slope and intercept of regression lines in terms of the original context.
 - d) *Determine the correlation, using technology, between two numerical unknowns, interpret the correlation, and describe the strengths and weaknesses of the correlation coefficient as a measure of linear association.
 - e) Interpolate using trends observed in scatter plots or fitted regression lines, and judge when extrapolating observed trends may be appropriate.
 - f) Given a set of data or scatter plot, choose the line of best fit and use the line to make predictions.
 - g) Recognize how linear transformations of one-variable data affect mean, median, mode, and range.
2. Describe the influence of outliers on correlation and on models for trend.
 - a) Identify unusual observations in scatter plots, and conjecture about the effect of such outliers on the strength of the association between the unknowns defining the scatter plot.
 - b) *Use technology to investigate and describe the influence outliers may have on a correlation coefficient, on the slope and intercept of a least-squares regression line, and on a median-fit line.

GEOMETRY

A percentage of instructional time for each core content area is suggested. The sum of the percentages totals less than 100% so that school districts can modify the emphasis and/or add additional content in response to local needs.

| | |
|---|------------|
| Geometry, Core Content A: Geometric Reasoning, Proof, and Representation | 25% |
|---|------------|

Students represent geometric figures and investigate a variety of relationships among them, form conjectures, and attempt to verify or reject the conjectures. They learn direct and indirect methods to prove or disprove conjectures within the axiomatic structure of Euclidean geometry.

| | |
|--|------------|
| Geometry, Core Content B: Similarity and Congruency | 25% |
|--|------------|

Students identify the properties and perform the mappings of rigid transformations and origin-centered dilations of figures in the plane. They discuss the connections between rigid transformations and origin-centered dilations and the relations of congruence and similarity, respectively. They develop and apply sufficient conditions for proving planar figures congruent or similar. They apply similarity to solve a variety of problems.

| | |
|--|------------|
| Geometry, Core Content C: Direct and Indirect Measurement | 25% |
|--|------------|

Students justify and apply formulas associated with measuring one-, two-, and three-dimensional geometric objects. They prove and apply the Pythagorean Theorem, right-triangle trigonometric ratios, and proportionality relationships in structuring and solving indirect measurement problems. Students extend their understanding of the real number system through solving problems in geometric situations.

| | |
|---|------------|
| Geometry, Core Content D: Conditional Probability and Independence | 10% |
|---|------------|

Students determine the sample space for multi-stage experiments. They distinguish between independent and dependent events and compute their probabilities. Students design and use simulations to solve problems.

NOTE: The order of the Core Content, Learning Goals, and Performance Indicators on the pages that follow does not prescribe a sequence of instruction. Sequencing should be determined by individual school districts based on their local curricula and instructional programs.

Geometry, Core Content A: Geometric Reasoning, Proof, and Representation**25%**

Students represent geometric figures and investigate a variety of relationships among them, form conjectures, and attempt to verify or reject the conjectures. They learn direct and indirect methods to prove or disprove conjectures within the axiomatic structure of Euclidean geometry.

Learning Goals and Performance (*Locally Assessed) Indicators:

1. Define, describe, and apply deductive reasoning by forming conjectures and attempting to verify or reject them through developing short sequences of geometric theorems within a local axiomatic system or by developing counterexamples.
 - a) Describe the structure of and relationships within an axiomatic system (undefined terms, defined terms, axioms/postulates, methods of reasoning, and theorems).
 - b) Form conjectures based on exploring geometric situations with and without technology.
 - c) Prove, directly or indirectly, that a valid mathematical statement is true or develop a counterexample to refute an invalid statement.
 - d) Identify flaws or gaps in the reasoning supporting an argument.
 - e) Formulate and investigate the validity of the converse, inverse, or contrapositive of a conditional statement.
2. Apply mathematical methods of proof to develop justifications for basic theorems of Euclidean geometry. Develop the triangle angle-sum and angle-measure theorems for polygons, and the triangle- and angle-inequality theorems.
 - a) Justify statements about angles formed by perpendicular lines and transversals of parallel lines.
 - b) Justify and apply properties of circles (e.g., perpendicularity of tangent and radius, angle inscribed in a circle).
 - c) Organize and present direct and indirect proofs using two-column, paragraph, and flow-chart formats.
3. Use a variety of representations to describe geometric objects and to analyze relationships among them.
 - a) Use coordinates and algebraic representations (e.g., distances, points that divide segments in specified ratios, slope) to describe and define figures.
 - b) Use nets, drawings (including isometric), vertex-edge graphs, models, and technologically created images to represent geometric objects from different perspectives and analyze relationships among them.

Geometry, Core Content B: Similarity and Congruency**25%**

Students identify the properties and perform the mappings of rigid transformations and origin-centered dilations of figures in the plane. They discuss the connections between rigid transformations and origin-centered dilations and the relations of congruence and similarity, respectively. They develop and apply sufficient conditions for proving planar figures congruent or similar. They apply similarity to solve a variety of problems.

Learning Goals and Performance (*Locally Assessed) Indicators:

1. Identify and apply transformations of figures in the coordinate plane and discuss the results of these transformations. Identify and justify congruence by establishing sufficient conditions and by finding a congruence-preserving rigid transformation between the figures. Solve problems involving congruence in a variety of contexts.
 - a) Identify and differentiate among sufficient conditions for congruence of triangles (SSS, SAS, ASA, AAS, and HL) and apply them to identify congruent triangles.
 - b) Represent translations, line reflections, rotations, and origin-centered dilations of objects in the coordinate plane by using sketches, coordinates, and function notation, and explain the effects of these transformations.
 - c) Recognize and identify corresponding parts of congruent and similar figures after transformation(s).
 - d) Use coordinate geometry and rigid transformations (reflections, translations, and rotations) to establish congruence of figures.
 - e) Apply geometric properties and relationships in solving multi-step problems in two and three dimensions.
2. Identify similar figures and justify similarity by establishing sufficient conditions and by finding an origin-centered dilation between the figures. Solve problems involving similarity in a variety of contexts.
 - a) Identify conditions for establishing similarity of triangles (SAS, SSS, AA), and apply them, noting that congruence is a special case of similarity.
 - b) Create a representation of a figure similar to a specified figure given their similarity ratio and use origin-centered dilations to describe and investigate similarities.

Geometry, Core Content C: Direct and Indirect Measurement**25%**

Students justify and apply formulas associated with measuring one-, two-, and three-dimensional geometric objects. They prove and apply the Pythagorean Theorem, right-triangle trigonometric ratios, and proportionality relationships in structuring and solving indirect measurement problems. Students extend their understanding of the real number system through solving problems in geometric situations.

Learning Goals and Performance (*Locally Assessed) Indicators:

1. Develop two- and three-dimensional formulas for measuring perimeter/circumference, area, and volume and apply these formulas and other geometric properties to solve problems involving simple and composite geometric objects.
 - a) Derive and use area formulas for quadrilaterals and regular polygons.
 - b) Link the surface area of prisms and cylinders to the sum of the areas of their bases and lateral surfaces using planar nets to illustrate and sum the relevant measures.
 - c) Develop the triangle angle-sum and angle-measure theorems for polygons, and the triangle- and angle-inequality theorems.
 - d) Identify and find the measures of angles formed by segments in three-dimensional figures, extending right triangle and isosceles/equilateral triangle relationships to study the planar faces of three-dimensional objects.
 - e) Apply and link formulas to solve problems involving area, perimeter, volume, and surface area of prisms, cylinders, pyramids, cones, spheres, and composite figures.
 - f) Apply the ratio of similitude to determine perimeter, area, and volume measurements of similar figures.
 - g) Determine arc lengths of circles, lengths of chords, lengths of tangent and secant segments, and areas of sectors of circles using proportions.
 - h) Use geometric models to represent and solve mathematical or real-world problems.
 - i) Justify and apply statements about central angles and angles formed by chords, tangents, and secants in circles and the measure of intercepted arcs.
2. Prove and apply the Pythagorean Theorem and its converse. Develop and apply the distance formula, properties of special right triangles, properties of proportions, and basic trigonometric ratios.
 - a) Apply the distance formula to determine the distance between points in the coordinate plane.
 - b) Apply properties of 30° - 60° - 90° and 45° - 45° - 90° triangles and the Pythagorean Theorem to determine triangle side lengths.
 - c) Prove and apply the Pythagorean Theorem and its converse.
 - d) Use properties of proportionality and similarity in solving problems in two- and three-dimensional settings.
 - e) *Apply the sine, cosine, and tangent trigonometric ratios to determine lengths and angle measures in right triangles.

Geometry, Core Content D: Conditional Probability and Independence**10%**

Students determine the sample space for multi-stage experiments. They distinguish between independent and dependent events and compute their probabilities. Students design and use simulations to solve problems.

Learning Goals and Performance (*Locally Assessed) Indicators:

1. Determine the sample space for multi-stage experiments (e.g., tossing three coins) using a variety of representations and employ systematic counting approaches to determine the number of possible outcomes. Distinguish between independent and dependent compound events, and compute their probabilities using a variety of representations and applying the multiplication rule for probability.
 - a) Use set notation to convey information and use Venn diagrams to illustrate given information about sets.
 - b) Determine the number of possible outcomes in the sample space for multi-stage experiments using a variety of representations (e.g., tree diagrams, lists) and systematic counting approaches including permutations, combinations, and the multiplication rule for counting (Fundamental Property of Counting).
 - c) Distinguish between independent and dependent compound events, and explain the idea of conditional probability using two-way tables and appropriate notation and terminology (e.g. union).
 - d) Use tables, trees, and geometric representations (area models) to describe and apply the multiplication rule for probability to compute probabilities for independent and for dependent compound events.
2. Develop, use, and interpret simulations to estimate probabilities for events where theoretical values are difficult or impossible to compute. Recognize that simulation results are likely to differ from one run of the simulation to the next, and that results of the simulation tend to converge as the number of trials increases.
 - a) *Describe and conduct a simulation by identifying the components and assumptions in a problem, selecting a device to generate chance outcomes, defining a trial, specifying the number of trials, and carrying out the simulation.
 - b) Summarize data from a simulation using appropriate graphical and numerical summaries, develop an estimate for the probability of an event where theoretical values are difficult or impossible to compute, and discuss the effect of the number of trials on the estimated probability of the event.

ALGEBRA II

A percentage of instructional time for each core content area is suggested. The sum of the percentages totals less than 100% so that school districts can modify the emphasis and/or add additional content in response to local needs.

| | |
|---|------------|
| Algebra II, Core Content A: Polynomial Expressions, Equations, and Functions | 30% |
| Students extend understanding of functions from linear settings to include polynomial functions (with a focus on quadratics), operations on these functions, and the solution of polynomial equations using complex numbers. They use polynomials to model situations with graphical and symbolic representations. They flexibly move among polynomial representations (e.g. factored form, standard form) to represent and discuss the qualitative behavior of the associated functions. | |

| | |
|--|------------|
| Algebra II, Core Content B: Exponential, Logarithmic, and Other Functions and Equations | 30% |
| Students develop exponential, logarithmic, and other nonlinear functions and equations (rational, radical, absolute value, and piecewise-defined) to represent, investigate, and solve mathematical and real-world problems. They maintain computational fluency with rational numbers by applying algorithms to operate on and simplify rational expressions. | |

| | |
|---|------------|
| Algebra II, Core Content C: Systems of Equations/Inequalities and Matrices | 15% |
| Students construct, solve, and interpret solutions of systems of equations and inequalities in two or more unknowns. They represent cross-categorized data in matrices and perform operations on matrices to model and interpret problem situations. They model problem situations with systems of equations and solve with technology. | |

| | |
|---|------------|
| Algebra II, Core Content D: Binomial Theorem and Probability | 10% |
| Students use the binomial expansion theorem and recognize the connection to Pascal's triangle, probability, and combinatorics. Students recognize a binomial probability setting, develop and graph the binomial probability distribution, and compute the probability distribution for a binomial count. | |

NOTE: The order of the Core Content, Learning Goals, and Performance Indicators on the pages that follow does not prescribe a sequence of instruction. Sequencing should be determined by individual school districts based on their local curricula and instructional programs.

Algebra II, Core Content A: Polynomial Expressions, Equations, and Functions**30%**

Students extend understanding of functions from linear settings to include polynomial functions (with a focus on quadratics), operations on these functions, and the solution of polynomial equations using complex numbers. They use polynomials to model situations with graphical and symbolic representations. They flexibly move among polynomial representations (e.g. factored form, standard form) to represent and discuss the qualitative behavior of the associated functions.

Learning Goals and Performance (*Locally Assessed) Indicators:

1. Use a variety of techniques to solve polynomial equations.
 - a) Use factoring (including sum and difference of cubes and other factorable polynomials) to transform expressions and solve equations.
 - b) Analyze and graph polynomial functions by identifying the intercepts, zeros, domain and range, turning points, and end behavior.
 - c) Apply the Fundamental Theorem of Algebra to determine the number of real and complex zeros of simple polynomial functions.
 - d) Apply the Fundamental Theorem of Algebra to determine the number of real and complex roots of a quadratic function.
 - e) Apply the composition of functions to model and solve problems, and explain the results.
2. Represent quadratic relationships in a variety of ways (including graphic, symbolic, and tabular) to interpret, solve problems, and verify solutions. Apply the quadratic formula to solve problems involving real and complex roots.
 - a) Translate between the standard form of a quadratic function, the vertex form, and the factored form; graph and interpret the meaning of each form.
 - b) Determine reasonable domain and range values for quadratic functions within a context, and test the reasonableness of solutions to quadratic equations (zeros of quadratic functions).
 - c) Graph quadratic equations and inequalities.
 - d) Recognize the relationships between the coefficients of a quadratic function and characteristics of its graph (e.g., shape, position, intercepts, zeros, maximum, minimum, lines of symmetry, vertex).
 - e) Solve quadratic equations and inequalities by factoring, completing the square and using the quadratic formula and justify solutions in terms of the original problem context.
3. Represent and apply properties of rational exponents and complex numbers.
 - a) Use the laws of exponents to simplify and evaluate numeric and algebraic expressions that contain rational exponents.
 - b) Define, plot, and compute with complex numbers.
 - c) Explain how whole, integer, rational, real, and complex numbers are related, and identify the number system(s) within which a given algebraic equation can be solved.
 - d) *Describe how the associative, commutative, and distributive properties of operations on real numbers extend to operations on complex numbers.

Algebra II, Core Content B: Exponential, Logarithmic, and Other Functions and Equations 30%

Students develop exponential, logarithmic, and other nonlinear functions and equations (rational, radical, absolute value, and piecewise-defined) to represent, investigate, and solve mathematical and real-world problems. They maintain computational fluency with rational numbers by applying algorithms to operate on and simplify rational expressions.

Learning Goals and Performance (*Locally Assessed) Indicators:

1. Represent geometric or exponential growth with exponential functions and equations, and apply such functions and equations to solve problems.
 - a) Simplify expressions with real (both rational and irrational) exponents and relate the expressions with rational exponents to equivalent radical expressions. Describe the effects of changes in the coefficient, base, and exponent on the growth described by an exponential function.
 - b) Analyze a problem situation modeled by an exponential function (e.g. exponential growth and decay, compound interest), formulate an equation or inequality, and solve the problem.
 - c) Graph and analyze the behavior of exponential functions.
2. Define logarithmic functions and use them to solve problems in mathematical or real-world situations.
 - a) Define a logarithm as a solution to an exponential equation, and recognize the inverse relationship between logarithmic functions and exponential functions, showing this relationship graphically.
 - b) Solve problems by applying properties of logarithms (for example, $\log_b xy = \log_b x + \log_b y$) to construct equivalent forms of a logarithmic expression.
 - c) Apply the inverse relationship between exponential and logarithmic functions to solve problems in context.
3. Interpret and represent rational and radical functions and solve rational and radical equations.
 - a) Model and solve problems using direct, inverse, joint, and combined variation.
 - b) Add, subtract, multiply, divide, and simplify rational expressions with simple polynomial denominators. Evaluate rational functions.
 - c) Model problem situations by constructing equations and inequalities based on rational and radical functions, solve using a variety of methods, and interpret solutions in terms of the problem situation.
 - d) Describe the graphs of rational and radical functions, including limitations on the domains and ranges, and asymptotic behavior.
 - e) Use properties of radicals to simplify radicals and radical expressions, to solve radical equations and to identify extraneous roots when they occur.
4. Interpret and model step and other piecewise-defined (i.e. linear and quadratic) functions, including absolute value functions.
 - a) Analyze a problem situation to determine or interpret reasonable domain and range values for piecewise-defined functions representing the situation.
 - b) Interpret, construct and apply step functions (e.g., greatest integer/floor) and other piecewise-defined functions, including absolute value functions, to model and solve problems.
 - c) Move flexibly between and among verbal, graphical, tabular, and symbolic representations of step functions and other piecewise-defined functions, including absolute value functions.

Algebra II, Core Content C: Systems of Equations/Inequalities and Matrices**15%**

Students construct, solve, and interpret solutions of systems of equations and inequalities in two or more unknowns. They represent cross-categorized data in matrices and perform operations on matrices to model and interpret problem situations. They model problem situations with systems of equations and solve with technology.

Learning Goals and Performance (*Locally Assessed) Indicators:

1. Construct, represent, solve, and interpret solutions of systems of equations and inequalities (including all combinations of linear and quadratic equations) in two unknowns.
 - a) Construct a system of equations or inequalities in two unknowns to represent a mathematical or real-world setting.
 - b) Analyze and explain the reasoning used to solve systems of linear equations and inequalities in two unknowns.
 - c) Solve a system of equations or inequalities in two unknowns using a variety of methods, and interpret the meaning of the solution.
2. Represent and interpret cross-categorized data in matrices, develop properties of matrix addition, and use matrix addition and its properties to solve problems.
 - a) *Represent numerical or relational data categorized by two or more unknowns in a matrix and label the rows and columns. Interpret the meaning of a particular entry in a matrix in terms of the labels of its row and column.
 - b) *Use matrix row and column sums to analyze data.
3. Multiply matrices, verify the properties of matrix multiplication, and use the matrix form for a system of linear equations to structure and solve systems consisting of two or three linear equations in two or three unknowns, respectively, with technology.
 - a) *Identify properties (e.g., associativity) that hold for matrix multiplication and multiply matrices to solve problems.
 - b) Construct a system of linear equations modeling a mathematical or real-world situation, and represent the system as a matrix equation ($Ax = b$), that is:

$$\begin{array}{l} ax + by = c \\ dx + ey = f \end{array} \Leftrightarrow \begin{bmatrix} a & b \\ d & e \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} c \\ f \end{bmatrix}$$

- c) *Solve a system consisting of two or more linear equations in two or more unknowns, respectively, by solving the related matrix equation $Ax = b$, using technology to find $x = A^{-1}b$.

Algebra II, Core Content D: Binomial Theorem and Probability**10%**

Students use the binomial expansion theorem and recognize the connection to Pascal's triangle, probability, and combinatorics. Students recognize a binomial probability setting, develop and graph the binomial probability distribution, and compute the probability distribution for a binomial count.

Learning Goals and Performance (*Locally Assessed) Indicators:

1. Develop facility with the binomial expansion theorem and its connections to Pascal's triangle, probability, and combinatorics. Recognize a binomial probability setting and compute the probability distribution for a binomial count.
 - a) *Use the binomial expansion theorem, and relate the expansion of $(a + b)^n$ with the possible outcomes of a binomial experiment and the n^{th} row of Pascal's triangle.
 - b) *Recognize a binomial probability setting, and develop and graph the probability distribution for a binomial count.

2. Identify settings in which the normal distribution may be useful, and describe characteristics of the normal distribution. Use graphical displays and the empirical rule to solve problems.
 - a) *Identify settings in which the normal distribution may be useful, and describe characteristics of a normal distribution including mean, standard deviation, and z-scores.
 - b) *Use graphical displays and the empirical rule to evaluate the appropriateness of the normal model for a given set of data, and use the empirical rule to estimate the probability that an event will occur in a specific interval that can be described in terms of whole numbers of standard deviations about the mean.

INTEGRATED MATHEMATICS I

A percentage of instructional time for each core content area is suggested. The sum of the percentages totals less than 100% so that school districts can modify the emphasis and/or add additional content in response to local needs.

Integrated Mathematics I, Core Content A: Expressions, Linear Equations, and Inequalities 25%

Students further develop ability to represent linear functions using symbolic notation and to solve linear equations and inequalities. They generate equivalent algebraic expressions and equations involving polynomials and rational expressions. They construct, represent, and solve linear equations, absolute value equations and linear inequalities. They determine the reasonableness of solution(s).

Integrated Mathematics I, Core Content B: Functions 15%

Students understand the concept of function and identify important features of functions and other relations. They represent functions using algebraic, graphical, tabular, and verbal representations (recognizing that tabular and graphical representations are usually only partial representations of the function). They move flexibly between and among representations. They apply the terminology and symbols associated with expressions, functions, and linear equations, including domain, range, slope, intercepts, independent, and dependent variables.

Integrated Mathematics I, Core Content C: Nonlinear Functions 20%

Students identify and classify nonlinear relationships. They represent mathematical or real-world problems using exponential and quadratic functions and solve equations involving these functions with a variety of techniques. They determine the reasonableness of the solution(s) and extend their understanding of the real number system through solving problems in algebraic situations.

Integrated Mathematics I, Core Content D: Geometric Reasoning and Relationships 15%

Students represent geometric objects and investigate a variety of relationships among them, form conjectures and attempt to verify or reject the conjectures. They develop and apply various methods of proving statements or disproving conjectures within the axiomatic structure of Euclidean geometry.

Integrated Mathematics I, Core Content E: Trend Lines and Correlation 10%

Students develop models for trends in bivariate data using both median-fit lines and least-squares regression lines. They use the correlation coefficient to measure linear association in scatterplots. Students examine the effects of outliers on the models for trend and on correlation. Students extend their understanding of data analysis through modeling and interpreting problems.

NOTE: The order of the Core Content, Learning Goals, and Performance Indicators on the pages that follow does not prescribe a sequence of instruction. Sequencing should be determined by individual school districts based on their local curricula and instructional programs.

Integrated Mathematics I, Core Content A: Expressions, Linear Equations, and Inequalities 25%

Students further develop ability to represent linear functions using symbolic notation and to solve linear equations and inequalities. They generate equivalent algebraic expressions and equations involving polynomials and rational expressions. They construct, represent and solve linear equations, absolute value equations and linear inequalities. They determine the reasonableness of solution(s).

Learning Goals and Performance (*Locally Assessed) Indicators:

1. Simplify expressions to generate equivalent forms to interpret and solve problems.
 - a) Factor common monomial factors from polynomials and factor quadratic polynomials.
 - b) Use the laws of exponents to simplify and evaluate numeric and algebraic expressions.
 - c) Represent square and cube roots using rational exponents.
 - d) Simplify square roots.
 - e) Simplify algebraic expressions, including those involving simple rational expressions [e.g., $2/x + 1/(1+x)$], and justify that two algebraic expressions, equations, or inequalities are equivalent.
 - f) Simplify absolute value expressions.
 - g) Add, subtract, multiply polynomials; divide a polynomial by a polynomial of equal or lower degree.
2. Solve and represent solutions of linear equations and inequalities. Understand the relationship between slopes of lines in the plane that are parallel, perpendicular, or neither.
 - a) Graph the line that represents solutions of a linear equation on the x-y coordinate plane.
 - b) Write equations and graph lines given particular information (e.g., two points on a line or slope and one point on the line).
 - c) Recognize and use different forms of linear equations including slope-intercept, point-slope, intercept, and general (standard) forms.
 - d) Identify the slope and x- and y-intercepts of a line given the equation of the line.
 - e) Recognize the relationship between the slopes of parallel lines and between perpendicular lines and use these relationships to identify and/or write equations for lines parallel or perpendicular to a given line.
 - f) Construct a linear equation or linear inequality to model a mathematical or real-world situation.
 - g) Solve linear equations and inequalities (one variable) using symbolic methods, graphs, tables, and technology.
 - h) Solve linear absolute value equations.

Integrated Mathematics I, Core Content B: Functions**15%**

Students understand the concept of function and identify important features of functions and other relations. They represent functions using algebraic, graphical, tabular, and verbal representations (recognizing that tabular and graphical representations are usually only partial representations of the function). They move flexibly between and among representations. They apply the terminology and symbols associated with expressions, functions, and linear equations, including domain, range, slope, intercepts, independent, and dependent variables.

Learning Goals and Performance (*Locally Assessed) Indicators:

- 1) Understand the definition of a function and identify important features of function and other relations using symbolic and graphical methods as appropriate.
 - a) Distinguish between functions and other relations defined symbolically, graphically, or in tabular form.
 - b) Represent a function with a symbolic expression, as a graph, in a table, and using words and make connections among these representations.
 - c) Find the domain and range of a function defined symbolically, graphically, or in a real-world context.
 - d) Compare linear and nonlinear functions by examining rates of change from verbal descriptions, table of values, graphical representations, and symbolic forms.

- 2) Describe and represent functions using appropriate notation and terminology and evaluate functions at given points.
 - a) Represent linear patterns or arithmetic sequences using verbal rules and symbolic expressions such as kx (proportional relationships) and $ax + b$ (more-general linear relationships).
 - b) Analyze a mathematical or real-world situation, determine whether a linear function can describe the situation, and if so, write the linear function.
 - c) Describe examples of situations represented by piecewise-linear functions.
 - d) Use functional notation and evaluate a function at a given point in its domain (e.g., *If $f(x) = 2x - 3$, find $f(-2)$.*

Integrated Mathematics I, Core Content C: Nonlinear Functions**20%**

Students identify and classify nonlinear relationships. They represent simple mathematical or real-world situations using exponential and quadratic functions and solve equations involving these functions with a variety of techniques. They determine the reasonableness of the solution(s) and extend their understanding of the real number system through solving problems in algebraic situations.

Learning Goals and Performance (*Locally Assessed) Indicators:

1. Identify nonlinear relationships and classify them as exponential, quadratic, or of the form $y = k/x$, based on rates of change in tables, symbolic forms, or graphical representations.
 - a) Identify nonlinear (exponential, quadratic, and equations of the form $y = k/x$) relationships in graphical or tabular displays through an examination of successive differences, ratios, symbolic forms, or graphical properties.
 - b) *Solve for terms in a geometric (exponential) sequence using given verbal rules or symbolic expressions (explicit and recursive).
 - c) Recognize exponential functions from their verbal descriptions and tabular, graphical or symbolic representations, and move flexibly between and among these representations.
 - d) Describe the effects of changes in the coefficient, base, and exponent on the growth described by an exponential function.
2. Represent and solve quadratic equations.
 - a) Represent a quadratic equation with a symbolic expression, as a graph, in a table, and with a description, and make connections among the representations.
 - b) Sketch the graph of a quadratic equation, describe the effects that changes in the parameters have on the graph, and interpret the x -intercepts as solutions to a quadratic equation.
 - c) Solve quadratic equations that can be factored as $(ax + b)(cx + d)$ where a , b , c , and d are integers using the zero-product property.
 - d) Solve quadratic equations of the form, $x^2 + bx + c = 0$, by completing the square.
 - e) *Solve quadratic equations using the quadratic formula.
3. Represent simple exponential functions using tables, symbolic forms, or graphical representations.
 - a) Sketch the graph for an exponential function of the form $y = ab^x$ where x is an integer.
 - b) Describe the effects that changes in the parameters a and b have on the graph of an exponential function of the form $y = ab^n$ where n is an integer.
 - c) Find and approximate solutions to exponential equations.
 - d) Distinguish among general forms for exponential equations ($y = b^x$, $y = a \cdot b^x$) and quadratic equations ($y = x^2$, $y = -x^2$, $y = ax^2$, $y = x^2 + c$, $y = ax^2 + c$) and describe how the values of a , b and c affect their graphical representations.

Integrated Mathematics I, Core Content D: Geometric Reasoning and Relationships**15%**

Students represent geometric objects and investigate a variety of relationships among them, form conjectures and attempt to verify or reject the conjectures. They develop and apply various methods of proving statements or disproving conjectures within the axiomatic structure of Euclidean geometry.

Learning Goals and Performance (*Locally Assessed) Indicators:

1. Use a variety of representations to describe geometric objects and to analyze relationships among them.
 - a) *Use coordinates and algebraic representations (e.g., distances, points that divide segments in specified ratios, slope) to describe and define figures.
 - b) *Use nets, drawings (including isometric), vertex-edge graphs, models, and technologically created images to represent geometric objects from different perspectives and analyze relationships among them.
2. Apply mathematical methods of proof to develop, test, and provide justifications, for conjectures involving relations of lines, angles, and figures.
 - a) *Describe the structure of and relationships within an axiomatic system (undefined terms, defined terms, axioms/postulates, methods of reasoning, and theorems).
 - b) *Recognize flaws or gaps in the reasoning supporting an argument.
 - c) *Test conjectures about angles, lines, bisectors, polygons (especially triangles and quadrilaterals), circles, and three-dimensional figures.
 - d) *Justify statements about angles formed by perpendicular lines and transversals of parallel lines.

Integrated Mathematics I, Core Content E: Trend Lines and Correlation**10%**

Students develop models for trends in bivariate data using both median-fit lines and least-squares regression lines. They use the correlation coefficient to measure linear association in scatterplots. Students examine the effects of outliers on the models for trend and on correlation. Students extend their understanding of data analysis through modeling and interpreting problems.

Learning Goals and Performance (*Locally Assessed) Indicators:

1. Model association of bivariate numerical data using tables and scatter plots and apply the correlation coefficient to measure and describe linear association. Apply both median-fit lines and least-squares regression lines to describe trends in bivariate data, when appropriate.
 - a) Compute the median-median line to model a relationship shown in a scatter plot, and interpret the slope and intercept in terms of the original context.
 - b) *Generate the least-squares regression line, using technology, to model a relationship shown in a scatter plot.
 - c) Interpret the slope and intercept of regression lines in terms of the original context.
 - d) *Determine the correlation, using technology, between two numerical unknowns, interpret the correlation, and describe the strengths and weaknesses of the correlation coefficient as a measure of linear association.
 - e) Interpolate using trends observed in scatter plots or fitted regression lines, and judge when extrapolating observed trends may be appropriate.
 - f) Given a set of data or scatter plot, choose the line of best fit and use the line to make predictions.
 - g) Recognize how linear transformations of one-variable data affect mean, median, mode, and range.
2. Describe the influence of outliers on correlation and on models for trend.
 - a) Identify unusual observations in scatter plots, and conjecture about the effect of such outliers on the strength of the association between the unknowns defining the scatter plot.
 - b) *Use technology to investigate and describe the influence outliers may have on a correlation coefficient, on the slope and intercept of a least-squares regression line, and on a median-fit line.

INTEGRATED MATHEMATICS II

A percentage of instructional time for each core content area is suggested. The sum of the percentages totals less than 100% so that school districts can modify the emphasis and/or add additional content in response to local needs.

| | |
|--|------------|
| Integrated Mathematics II, Core Content A: Systems of Linear Equations and Matrices | 25% |
|--|------------|

Students represent relationships that can be modeled by a system of linear equations and solve the system using a variety of methods and representations. They represent and interpret data and systems of equations through matrix representations, using addition and multiplication of matrices as appropriate. They use matrix equations and inverses, where they exist, to find solutions to systems of equations using technology. Students extend their understanding of the real number system through solving problems in algebraic situations.

| | |
|--|------------|
| Integrated Mathematics II, Core Content B: Geometric Proof, Similarity, and Transformations | 30% |
|--|------------|

Students apply and analyze transformations of figures in the coordinate plane using properties of similarity and congruency and apply mathematical methods of proof and reasoning to form or verify conjectures. They learn direct and indirect methods to prove or disprove conjectures and apply these methods to solve problems involving congruence, similarity, rigid transformations, and origin-centered dilations of figures in the plane.

| | |
|---|------------|
| Integrated Mathematics II, Core Content C: Direct and Indirect Measurement | 20% |
|---|------------|

Students justify and apply the measurement formulas associated with one-, two-, and three-dimensional geometric objects. They extend their understanding of the real number system through solving problems in geometric situations.

| | |
|--|------------|
| Integrated Mathematics II, Core Content D: Conditional Probability and Independence | 10% |
|--|------------|

Students determine the sample space for multi-stage experiments. They distinguish between independent and dependent events and compute their probabilities. Students design and use simulations to solve problems.

NOTE: The order of the Core Content, Learning Goals, and Performance Indicators on the pages that follow does not prescribe a sequence of instruction. Sequencing should be determined by individual school districts based on their local curricula and instructional programs.

Integrated Mathematics II, Core Content A: Systems of Linear Equations and Matrices **25%**

Students represent relationships that can be modeled by a system of linear equations and solve the system using a variety of methods and representations. They represent and interpret data and systems of equations through matrix representations, using addition and multiplication of matrices as appropriate. They use matrix equations and inverses, where they exist, to find solutions to systems of equations using technology. Students extend their understanding of the real number system through solving problems in algebraic situations.

Learning Goals and Performance (*Locally Assessed) Indicators:

1. Represent relationships that can be modeled by a system of linear equations and solve the system using a variety of methods and representations.
 - a) Construct a system of linear equations modeling a mathematical or real-world situation.
 - b) Graph a system of linear equations and indicate possible solutions (intersection points).
 - c) Analyze, solve, and explain the reasoning used to solve a system of linear equations and describe the nature of the solutions (no solution, one solution, infinitely many solutions).
2. Represent and interpret cross-categorized data in matrices, develop properties of matrix addition, and use matrix addition and its properties to solve problems.
 - a) Represent numerical or relational data categorized by two or more unknowns in a matrix and label the rows and columns. Interpret the meaning of a particular entry in a matrix in terms of the labels of its row and column.
 - b) Develop the properties of matrix addition, and add and subtract matrices to solve problems.
3. Multiply matrices, verify the properties of matrix multiplication, and use the matrix form for a system of linear equations to structure and solve systems consisting of two or three linear equations in two or three unknowns, respectively, with technology.
 - a) Verify the properties of matrix multiplication and multiply matrices to solve problems.
 - b) Construct a system of linear equations modeling a mathematical or real-world situation, and represent the system as a matrix equation ($Ax = b$), that is:

$$\begin{array}{l} ax + by = c \\ dx + ey = f \end{array} \Leftrightarrow \begin{bmatrix} a & b \\ d & e \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} c \\ f \end{bmatrix}$$

- c) Solve a system consisting of two or more linear equations in two or more unknowns, respectively, by solving the related matrix equation $Ax = b$, using technology to find $x = A^{-1}b$.

Integrated Mathematics II, Core Content B: Geometric Proof, Similarity, and Transformations 30%

Students apply and analyze transformations of figures in the coordinate plane using properties of similarity and congruency and apply mathematical methods of proof and reasoning to form or verify conjectures. They learn direct and indirect methods to prove or disprove conjectures and apply these methods to solve problems involving congruence, similarity, rigid transformations, and origin-centered dilations of figures in the plane.

Learning Goals and Performance (*Locally Assessed) Indicators:

1. Apply mathematical methods of proof to develop justifications for basic theorems of Euclidean geometry.
 - a) Form conjectures based on exploring geometric situations with and without technology.
 - b) Prove, directly or indirectly, that a valid mathematical statement is true or develop a counterexample to refute an invalid statement.
 - c) Formulate and investigate the validity of the converse, inverse, or contrapositive of a conditional statement.
 - d) Organize and present direct and indirect proofs using two-column, paragraph, and flow-chart formats.
2. Identify congruent figures and justify these congruencies by establishing sufficient conditions and by finding a congruence-preserving transformation between the figures. Solve problems involving congruence in a variety of contexts.
 - a) Analyze figures in terms of their symmetries using the concepts of reflection, rotation, and translation and combinations of these.
 - b) Compare and contrast equality, congruence, and similarity.
 - c) Identify and differentiate among sufficient conditions for congruence of triangles (SSS, SAS, ASA, AAS, and HL).
 - d) Use coordinate geometry and rigid transformations (reflections, translations, and rotations) to establish congruence of figures.
3. Identify and apply transformations of figures in the coordinate plane and discuss the results of these transformations.
 - a) Represent translations, line reflections, rotations, and origin-centered dilations of objects in the coordinate plane by using sketches, coordinates, and function notation, and explain the effects of these transformations.
 - b) Recognize and identify corresponding parts of congruent and similar figures after transformation.
4. Identify similar figures and justify similarity by establishing sufficient conditions and by finding a similarity-preserving rigid transformation or origin-centered dilation between the figures. Solve problems involving similarity in a variety of contexts.
 - a) Identify conditions (SAS, SSS, and AA) for establishing similarity of triangles, and apply them, noting that congruence is a special case of similarity.
 - b) Use similarity to calculate the measures of corresponding parts of similar figures, and apply similarity in a variety of problem-solving contexts within mathematics and other disciplines.
 - c) Create a representation of a figure similar to a specified figure given their similarity ratio and use origin-centered dilations to describe and investigate similarities.
 - d) Use similar triangles to demonstrate that the rate of change associated with any pair of points on a line is the same.

Integrated Mathematics II, Core Content C: Direct and Indirect Measurement**20%**

Students justify and apply the measurement formulas associated with one-, two-, and three-dimensional geometric objects. They extend their understanding of the real number system through solving problems in geometric situations.

Learning Goals and Performance (*Locally Assessed) Indicators:

1. Develop two- and three-dimensional measurement formulas for perimeter/circumference, area, and volume and apply these formulas and other geometric properties to solve problems involving simple and composite geometric objects.
 - a) Derive and use area formulas for quadrilaterals and regular polygons.
 - b) Link the surface area of prisms and cylinders to the sum of the areas of their bases and lateral surfaces using planar nets to illustrate and sum the relevant measures.
 - c) Identify and find the measures of angles formed by segments in three-dimensional figures, extending right triangle and isosceles/equilateral triangle relationships to study the planar faces of three-dimensional objects.
 - d) Apply and relate formulas and solve problems involving area, perimeter, volume, and surface area of pyramids, cones, spheres, and composite figures.
 - e) Determine arc lengths of circles, lengths of chords, and areas of sectors of circles using proportions.
 - f) Develop the triangle angle-sum and angle-measure theorems for polygons, and the triangle- and angle-inequality theorems.
 - g) Justify and apply statements about central angles and angles formed by chords, tangents, and secants in circles and the measure of intercepted arcs.

Integrated Mathematics II, Core Content D: Conditional Probability and Independence **10%**

Students determine the sample space for multi-stage experiments. They distinguish between independent and dependent events and compute their probabilities. Students design and use simulations to solve problems.

Learning Goals and Performance (*Locally Assessed) Indicators:

1. Determine the sample space for multi-stage experiments (e.g., tossing three coins) using a variety of representations and employ systematic counting approaches to determine the number of possible outcomes. Distinguish between independent and dependent compound events, and compute their probabilities using a variety of representations and applying the multiplication rule for probability.
 - a) Use set notation to convey information and use Venn diagrams to illustrate given information about sets.
 - b) Determine the number of possible outcomes in the sample space for multi-stage experiments using a variety of representations (e.g., tree diagrams, lists) and systematic counting approaches including permutations, combinations, and the multiplication rule for counting (Fundamental Property of Counting).
 - c) Distinguish between independent and dependent compound events, and explain the idea of conditional probability using two-way tables and appropriate notation and terminology (e.g. union)
 - d) Use tables, trees, and geometric representations (area models) to describe and apply the multiplication rule for probability to compute probabilities for independent and for dependent compound events.
2. Develop, use, and interpret simulations to estimate probabilities for events where theoretical values are difficult or impossible to compute. Recognize that simulation results are likely to differ from one run of the simulation to the next, and that results of the simulation tend to converge as the number of trials increases.
 - a) *Describe and conduct a simulation by identifying the components and assumptions in a problem, selecting a device to generate chance outcomes, defining a trial, specifying the number of trials and carrying out the simulation.
 - b) Summarize data from a simulation using appropriate graphical and numerical summaries, develop an estimate for the probability of an event where theoretical values are difficult or impossible to compute, and discuss the effect of the number of trials on the estimated probability of the event.

INTEGRATED MATHEMATICS III

A percentage of instructional time for each core content area is suggested. The sum of the percentages totals less than 100% so that school districts can modify the emphasis and/or add additional content in response to local needs.

Integrated Mathematics III, Core Content A: Polynomial Expressions, Equations, and Functions 30%

Students extend understanding of functions from linear settings to include polynomial functions, systems, operations on these functions, and the solution of polynomial equations using complex numbers. They use polynomials (with a focus on quadratics) to model situations with graphical and symbolic representations. They flexibly move among polynomial representations (e.g. factored form, standard form) to represent and discuss the qualitative behavior of the associated functions.

Integrated Mathematics III, Core Content B: Exponential, Logarithmic, and Other Functions and Equations 25%

Students develop exponential, logarithmic, and other nonlinear functions and equations (rational, radical, absolute value, and piecewise-defined) to represent, investigate, and solve mathematical and real-world problems. They extend their understanding of the real number system through solving problems in algebraic situations.

Integrated Mathematics III, Core Content C: Sequences and Recursion 10%

Students analyze and represent sequences and series and investigate how recursive relationships and their associated sequences can model the long-term behavior of situations involving sequential change. Students extend their understanding of the real number system through solving problems in algebraic situations.

Integrated Mathematics III, Core Content D: Indirect Measurement 10%

Students prove and apply the Pythagorean Theorem, right-triangle trigonometric ratios, and proportionality relationships in structuring and solving indirect measurement problems. They extend their understanding of the real number system through solving problems in geometric situations.

Integrated Mathematics III, Core Content E: Binomial Theorem and Probability 10%

Students use the binomial expansion theorem and recognize the connection to Pascal's triangle, probability, and combinatorics. Students recognize a binomial probability setting, develop and graph the binomial probability distribution, and compute the probability distribution for a binomial count. They extend their understanding of the real number system through problem solving situations.

NOTE: The order of the Core Content, Learning Goals, and Performance Indicators on the pages that follow does not prescribe a sequence of instruction. Sequencing should be determined by individual school districts based on their local curricula and instructional programs.

Integrated Mathematics III, Core Content A: Polynomial Expressions, Equations and Functions 30%

Students extend understanding of functions from linear settings to include polynomial functions, systems, operations on these functions, and the solution of polynomial equations using complex numbers. They use polynomials (with a focus on quadratics) to model situations with graphical and symbolic representations. They flexibly move among polynomial representations (e.g. factored form, standard form) to represent and discuss the qualitative behavior of the associated functions.

Learning Goals and Performance (*Locally Assessed) Indicators:

1. Use a variety of techniques to solve polynomial equations.
 - a) Use factoring (including sum and difference of cubes and other factorable polynomials) to transform expressions and solve equations.
 - b) Analyze and graph polynomial functions by identifying the intercepts, zeros, domain and range, turning points, and end behavior.
 - c) Apply the Factor Theorem to solve polynomial equations and the Remainder Theorem to evaluate polynomial functions.
 - d) Apply the Fundamental Theorem of Algebra to determine the number of real and complex roots of a quadratic function.
 - e) Apply the composition of functions to model and solve problems, and explain the results.
2. Represent quadratic relationships in a variety of ways (including graphic, symbolic, and tabular) to interpret, solve problems, and verify solutions. Apply the quadratic formula to solve problems involving real and complex roots.
 - a) Translate between the standard form of a quadratic function, the vertex form, and the factored form; graph and interpret the meaning of each form.
 - b) Determine reasonable domain and range values for quadratic functions within a context, and test the reasonableness of solutions to quadratic equations (zeros of quadratic functions).
 - c) Graph quadratic equations and inequalities.
 - d) Recognize the relationships between the coefficients of a quadratic function and characteristics of its graph (e.g., shape, position, intercepts, zeros, maximum, minimum, lines of symmetry, vertex).
 - e) Solve quadratic equations and inequalities by factoring, completing the square and using the quadratic formula and justify solutions in terms of the original problem context.
3. Construct, solve, and interpret solutions of systems of equations and inequalities (including all combinations of linear and quadratic equations) in two unknowns.
 - a) Construct a system of equalities or inequalities in two unknowns to represent a mathematical or real-world problem.
 - b) *Analyze and explain the reasoning used to solve systems of linear equations and inequalities in two unknowns.
 - c) Solve a system of equalities or inequalities in two unknowns using a variety of methods, and interpret the meaning of the solution.
4. Represent and apply properties of rational exponents and complex numbers.
 - a) Use the laws of exponents to simplify and evaluate numeric and algebraic expressions that contain rational exponents.
 - b) Define, plot, and compute with complex numbers.
 - c) Explain how whole, integer, rational, real, and complex numbers are related, and identify the number system(s) within which a given algebraic equation can be solved.
 - d) *Describe how the associative, commutative, and distributive properties of operations on real numbers extend to operations on complex numbers.

Integrated Mathematics III, Core Content B: Exponential, Logarithmic, and Other Functions and Equations**25%**

Students develop exponential, logarithmic, and other nonlinear functions and equations (rational, radical, absolute value, and piecewise-defined) to represent, investigate, and solve mathematical and real-world problems. They extend their understanding of the real number system through solving problems in algebraic situations.

Learning Goals and Performance (*Locally Assessed) Indicators:

1. Represent geometric or exponential growth with exponential functions and equations, and apply such functions and equations to solve problems.
 - a) Simplify expressions with real (both rational and irrational) exponents and relate the expressions with rational exponents to equivalent radical expressions. Describe the effects of changes in the coefficient, base, and exponent on the growth described by an exponential function.
 - b) Analyze a problem situation modeled by an exponential function (e.g. exponential growth and decay, compound interest), formulate an equation or inequality, and solve the problem.
 - c) Graph and analyze the behavior of exponential functions.
 - d) *Identify any points of intersection of the graph exponential equation of the form $y = ab^x$ and the graph of a line of the form $y = k$ with and without technology and relate the points of intersection to the solutions of the exponential equation $y = ab^x$.
2. Define logarithmic functions and use them to solve problems.
 - a) Define a logarithm as a solution to an exponential equation, and recognize the inverse relationship between logarithmic functions and exponential functions, showing this relationship graphically.
 - b) Solve problems by applying properties of logarithms (for example, $\log_b xy = \log_b x + \log_b y$) to construct equivalent forms of a logarithmic expression.
 - c) Apply the inverse relationship between exponential and logarithmic functions to solve problems.
3. Interpret and represent rational and radical functions and solve rational and radical equations.
 - a) Model and solve problems using direct, inverse, joint, and combined variation.
 - b) Add, subtract, multiply, divide, and simplify rational expressions with linear and quadratic denominators. Evaluate rational functions.
 - c) Model problem situations by constructing equations and inequalities based on rational and radical functions, solve using a variety of methods, and interpret solutions in terms of the problem situation.
 - d) Describe the graphs of rational and radical functions, including limitations on the domains and ranges, and asymptotic behavior.
 - e) Use properties of radicals to compute and simplify radicals, radical expressions, to solve radical equations and to identify extraneous roots when they occur.
4. Interpret and model step and other piecewise-defined (i.e. linear and quadratic) functions, including absolute value functions.
 - a) Analyze a problem situation to determine or interpret reasonable domain and range values for piecewise-defined functions representing the situation.
 - b) Interpret, construct, and apply step functions (e.g., greatest integer/floor) and other piecewise-defined functions, including absolute value functions, to model and solve problems.
 - c) *Move flexibly between and among verbal, graphical, tabular, and symbolic representations of step functions and other piecewise-defined functions, including absolute value functions.

Integrated Mathematics III, Core Content C: Sequences and Recursion**10%**

Students analyze and represent sequences and series and investigate how recursive relationships and their associated sequences can model the long-term behavior of situations involving sequential change. Students extend their understanding of the real number system through solving problems in algebraic situations.

Learning Goals and Performance (*Locally Assessed) Indicators:

1. Categorize sequences as arithmetic, geometric, or neither and develop formulas for the general terms and sums related to arithmetic and geometric sequences.
 - a) *Investigate the rate of change found in sequences, and use it to characterize sequences as arithmetic, geometric, or neither.
 - b) Develop the general term for arithmetic and geometric sequences and develop methods for calculating sums of terms for finite arithmetic and geometric sequences and the sum of a convergent infinite geometric series.
2. Develop and apply recursive relationships for modeling and investigating patterns in the long-term behavior of their associated sequences.
 - a) Develop and apply recursive relationships for arithmetic and for geometric growth situations.
 - b) Generate or construct sequences from given recursive relationships modeling patterns found in mathematics and in other disciplines.
 - c) *Analyze the long-term behavior of a recursive relationship, with and without technology.

Integrated Mathematics III, Core Content D: Indirect Measurement**10%**

Students prove and apply the Pythagorean Theorem, right-triangle trigonometric ratios, and proportionality relationships in structuring and solving indirect measurement problems. They extend their understanding of the real number system through solving problems in geometric situations.

Learning Goals and Performance (*Locally Assessed) Indicators:

1. Prove and apply the Pythagorean Theorem and its converse. Develop and apply the distance formula, properties of special right triangles, properties of proportions, and the basic trigonometric ratios.
 - a) Apply the distance formula to determine the distance between points in the coordinate plane.
 - b) *Apply properties of 30° - 60° - 90° and 45° - 45° - 90° triangles and the Pythagorean Theorem to determine triangle side lengths.
 - c) Prove and apply the Pythagorean Theorem and its converse.
 - d) Use properties of proportionality and similarity in solving problems in two- and three-dimensional settings.
 - e) Apply the sine, cosine, and tangent trigonometric ratios to determine lengths and angle measures in right triangles.

Integrated Mathematics III, Core Content E: Binomial Theorem and Probability**10%**

Students use the binomial expansion theorem and recognize the connection to Pascal's triangle, probability, and combinatorics. Students recognize a binomial probability setting, develop and graph the binomial probability distribution, and compute the probability distribution for a binomial count. They extend their understanding of the real number system through problem solving situations.

Learning Goals and Performance (*Locally Assessed) Indicators:

1. Develop facility with the binomial expansion theorem and its connections to Pascal's triangle, probability, and combinatorics. Recognize a binomial probability setting and compute the probability distribution for a binomial count.
 - a) *Use the binomial expansion theorem, and relate the expansion of $(a + b)^n$ with the possible outcomes of a binomial experiment and the n^{th} row of Pascal's triangle.
 - b) *Recognize a binomial probability setting, and develop and graph the probability distribution for a binomial count.
2. Identify settings in which the normal distribution may be useful, and describe characteristics of the normal distribution. Use graphical displays and the empirical rule to solve problems.
 - a) *Identify settings in which the normal distribution may be useful, and describe characteristics of a normal distribution including mean, standard deviation, and z-scores.
 - b) *Use graphical displays and the empirical rule to evaluate the appropriateness of the normal model for a given set of data, and use the empirical rule to estimate the probability that an event will occur in a specific interval that can be described in terms of whole numbers of standard deviations about the mean.

NOTES:

Core content, learning goals, and performance indicators for two additional courses (Precalculus and Integrated IV) are under development.

A glossary of key terms related to this document is under development. The glossary previously prepared for the 2004 GLE document is online at the DESE website (see: <http://dese.mo.gov/divimprove/curriculum/math/>). A revised version, adding key terms used in this document, will be posted soon.

DRAFT